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On the stable growth index for a grade-structured manpower system

V. U. Ekhosuehi[a*], O. Enagbonma[b] and A. A. Osagiede[c]

[a]*Department of Mathematics (Statistics Research Group), University of Benin, Benin City, Nigeria*; [b]*Department of Physical Sciences (Mathematics Option), Benson Idahosa University, Benin City, Nigeria*; [c]*Department of Mathematics, University of Benin, Benin City, Nigeria*

This paper develops a stable growth index for a manpower system using the stocks and flows of the system. This is achieved by modelling the dynamics of the system using a homogeneous Markov chain and the resulting system of equations is solved by applying the method of least squares. The utility of the stable growth index is illustrated with a university-faculty manpower structure and the results show that the manpower stocks expand over time within an acceptable control interval defined by the Individual-Chart-Moving-Range.

Keywords: employment; Individual-Chart-Moving-Range; manpower system; Markov chain; stable growth index.

1. Introduction

Rising unemployment is one of the major problems confronting developing economies (Elikwu, 2003). This problem has been aggravated by the government's concern on opening additional higher institutions with little or no plan to match the graduate output with the manpower demand (Jhingan, 2003). Consequently, there have been brain drain, increase in crime, high dependency ratio, wastage of human and material resources, job scam, and a fall in standard of living (Ekhosuehi and Osagiede, 2007). The perturbing issue is hinged on the recruitment drive of organizations vis-á-vis the wastage over time. Even so recruitment is affected by several factors. According to McClean (1976a), the number and frequency of recruits to an organization are influenced by the following factors:

(1) The stage of development of the organization, i.e. whether it is newly formed, and therefore expanding, or an established firm in steady state.
(2) The size of the organization.
(3) The pool of suitably qualified people available for recruitment.
(4) The economic environment, i.e. is the economy booming and therefore expansion feasible, or is there an economic recession which puts paid to any immediate thoughts of development?

These factors have led to the formulation of assumptions in the model description of recruits to a manpower system (for instance, McClean, 1976a; Woodward, 1983). Such assumptions usually apply to factors (1) and (2), because factors (3) and (4) are external to the organization.

There is a need to examine whether an organization recruits more than the wastage in the system. To examine this issue, organizations may be classified into two groups according to their recruitment drive, viz. the organizations on the vanguard for employment (e.g., a newly formed organization, an expanding organization, etc.) and the organizations clogging up on employment (e.g., an established firm in steady state). Organizations on the vanguard for employment are those that recruit more than the wastage over time, while organizations

*Corresponding author. Email: virtue.ekhosuehi@uniben.edu

clogging up on employment are those characterized by contraction of the workforce. Studies on organizations with contracting workforce have revealed the possibility of negative entries in the recruitment vector (Hopkins, 1980; Bartholomew et al., 1991). To avoid the trauma arising from a contracting workforce size (via retrenchment), Osagiede et al. (2007) proposed the policy of delayed promotion (stagnation) for organizations. McClean (1976a) developed both deterministic and stochastic models for company growth by assuming that recruitment is described as either being Poisson or dependent on the size of the company. The models were formulated in terms of the manpower stocks with an exponential growth rate. The work suggested that a more realistic model should be constructed wherein the manpower stocks are stratified according to their grades and that promotion should be taking into consideration. These suggestions form the basis for the stable growth index in this paper. Later on, McClean (1976b) modelled the growth of a firm by considering the number of employees and its capital resources as interacting processes. These models of McClean are in continuous-time. However we do not focus on continuous-time models. This is because employee's growth is measured at discrete time points.

The thematic focus of this paper is to develop a stable growth index for manpower systems using manpower stocks and flows. The use of the terms 'stocks' and 'flows' for the dynamics of a manpower system is consistent with Vassiliou (1981) and Bartholomew et al. (1991). We consider a stable manpower system stratified into discrete finite non-overlapping states defined in terms of grades.

DEFINITION 1.1 *(Vassiliou, 1981): A manpower system is called stable if for its sequence* $\{\boldsymbol{n}(t)\}$ *of stock vectors consisting of expected grade sizes,* $\boldsymbol{n}(t+1) = \langle\vartheta\rangle\, \boldsymbol{n}(t)$ *holds with a positive scalar* $\langle\vartheta\rangle$ *called the stable growth factor per unit time.*

The notations used in Definition 1.1 are consistent with the ones used in this paper. Hereafter, we shall refer to the positive scalar $\langle\vartheta\rangle$ as the stable growth index. This stable growth index should not be equated to the one arising from the difference equation of the form $Y_{t+1} = \langle\vartheta\rangle\, Y_t$ with $|\langle\vartheta\rangle| < 1$ if the process Y_t is stable. For instance, the relation $-1 < \langle\vartheta\rangle < 0$ does not hold for a manpower system. Moreover, in a non-contracting manpower system $\langle\vartheta\rangle \geq 1$. This can be verified as follows. Let the growth rate per unit time be $\alpha \geq 0$ for the stocks. Then

$$\alpha = \frac{\mathbf{n}(t+1) - \mathbf{n}(t)}{\mathbf{n}(t)((t+1) - t)}.$$

This simplifies to $\mathbf{n}(t+1) = (1+\alpha)\mathbf{n}(t)$. It follows that $\langle\vartheta\rangle = 1 + \alpha \geq 1$.

The setting we consider is the system where the recruitment plan is such that recruitment is done to replace wastage and to achieve the desired growth and the dynamics of the environment is described by a homogenous Markov chain (Woodward, 1983; Even-Dar et al., 2009). The desired growth herein is the stable growth. To make our discussion clearer to a wider audience, we give the following definition.

DEFINITION 1.2 *(Ibe, 2009): A Markov chain is a discrete-state Markov process* $\{X_t \,|t \in \Im\}$ *where*

$$Prob\,\{X_{t+1} = s_j \,|X_0 = s_1, \cdots, X_t = s_i\} = Prob\,\{X_{t+1} = s_j \,|X_t = s_i\},$$

$i, j = 1, 2, \cdots, k.$

The Markov chain described in Definition 1.2 is called a non-homogeneous discrete-time Markov chain. If $Prob\,\{X_{t+1} = s_j \,|X_t = s_i\} = p_{ij}$, then the Markov chain is said to be homogeneous.

We propose a new growth index for the manpower system and use the Individual-Chart-Moving-Range (ICMR) framework to adjudge the growth index. This paper was initially aimed at developing a stable growth index that will give us an insight into whether or not an organization is on the vanguard for employment. In the course of doing this, we realized that there are several approaches to finding a growth index for a system. What follows from this realization is a comparison of our proposed stable growth index with the ones in the literature (for instance, Osagiede and Ekhosuehi, 2006; Zanakis and Maret, 1980). This comparison is made by setting up control limits for the manpower stocks generated by the growth indices using the ICMR. Values that lie within the control limits imply that the growth index generating them is acceptable (i.e. in control); otherwise, it is unacceptable. The growth indices in the literature were developed using manpower stocks. This paper contributes to the literature by developing a stable growth index that utilizes both manpower stocks and flows. This contribution is in line with the future research direction suggested by McClean (1976a).

2. Materials and method

The model developed in this section follows from the ideas of Bartholomew et al. (1991) and Vassiliou (1981). These studies have created a platform for a manpower system to be stable and for recruitment to the system to be considered as being done to replace wastage and to achieve the desired growth. The section is divided into three subsections. Subsection 2.1 contains the existing growth indices. Subsection 2.2 develops the new stable growth index. The use of the new stable growth index as an employment index is discussed in Subsection 2.3.

2.1 *Existing growth indices*

Consider a $k-$graded manpower system with $S = \{1, 2, \cdots, k\}$ being the set of non-overlapping states. The state of the system at time t is represented by the row vector

$$\mathbf{n}(t) = [n_1(t), n_2(t), \cdots, n_k(t)],$$

where $n_i(t)$ is the number of individuals of the system in state i at time t. The vector $\mathbf{n}(t)$ is called the structure of the system at time t. The total stock of the system at time t is

$$N(t) = \sum_{i=1}^{k} n_i(t).$$

The system is assumed to grow at a desired rate per unit time called the stable growth rate. Let α denote the stable growth rate per unit time. Instances where this growth rate, α, is unknown to the researcher abound in the literature (Hopkins, 1980; Osagiede and Omosigho, 2004). Rather than using arbitrary values, the growth rate may be determined from historical dataset by solving the difference equation $\mathbf{n}(t+1) = (1+\alpha)\mathbf{n}(t)$ for α. Osagiede and Ekhosuehi (2006) provided an estimator, $\hat{\alpha}$, for the stable growth rate based on the individual stocks and the duration of the dataset. The estimator is given as

$$\hat{\alpha} = \exp\left(\frac{12\sum_{t=0}^{T}(t+1)\ln N(t) - 6(T+2)\ln\prod_{t=0}^{T} N(t)}{(T+1)((T+1)^2-1)}\right). \tag{2.1}$$

Another approach to obtaining the stable growth rate is in terms of the total gain or loss (Vassiliou, 1976; Zanakis and Maret, 1980; Kim, 1985). This is done by periodically computing the total gain or loss from a dataset and the trend over time is obtained by taking the average of the periodic growth rate. The growth rate, denoted by $\bar{\bar{\alpha}}$, is given as

$$\bar{\bar{\alpha}} = \frac{1}{T}\sum_{t=1}^{T}\frac{\Delta N(t)}{N(t-1)}, \tag{2.2}$$

where $\Delta N(t) = N(t) - N(t-1)$ denotes the change in total stock at time t.

The use of the total stock has the limitation of not accounting for the career prospects of the workforce. This is because promotion rates are not considered. Bartholomew et al. (1991) had earlier posited that the stock vector provides a 'snapshot' of the system but tells us nothing directly about change over time. This position of Bartholomew et al. (1991) coupled with the fact that the evolution of the sequence of stock vectors, $\{\mathbf{n}(t)\}$, is affected by flows such as promotion, wastage and recruitment (Setlhare, 2007) limits the use of the formulas in Eq. (2.1) and Eq. (2.2) as stable growth rate indices. Against this background, it becomes pertinent to develop a new stable growth rate index which utilizes the stocks and flows in an organizational system.

2.2 *The model*

We construct a stable growth index that incorporates the flows and recruits in a manpower system. We shall treat flows associated with transitions within and losses from the system in the time interval $(t, t+1)$, $t \in \Im$, $\Im = \{0, 1, 2, \cdots, T\}$, as though they took place at the starting-point. We assume that the operational policy of the system will persist into the future. Assume a state 0 which denotes the environment outside the organizational system. The flows are assumed to be governed by transition probabilities and the grades are assumed to be independent with respect to those probabilities. Flows are considered as a random variable with a multinomial distribution (Woodward, 1983). The distribution of individual movement in the system is given as:

$$P(n_{i0}(t), n_{i1}(t), \ldots, n_{ik}(t)) = \frac{\left(\sum_{j=0}^{k} n_{ij}(t)\right)!}{\prod_{j=0}^{k}(n_{ij}(t))!}\prod_{j=0}^{k}(p_{ij}(t))^{n_{ij}(t)}, \tag{2.3}$$

where $n_{ij}(t)$ is the number of individuals moving from grade i to grade j, $p_{ij}(t)$ is the transition probability from grade i to grade j in period t, and $j \in \{0\} \cup S$. We also assume that individuals move independently and with identical probabilities which do not vary over time (Bartholomew et al., 1991). The transition probabilities are estimated using the maximum likelihood method (Zanakis and Maret, 1980) as

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{\sum_{j=0}^{k} n_{ij}(t)}, \tag{2.4}$$

where $\sum_{j=0}^{k} n_{ij}(t) > 0, n_{ij}(t) \geq 0$. Suppose that individuals move independently and with identical probabilities which do not vary over time. Then the transition probability estimate becomes

$$\hat{p}_{ij} = \frac{\sum_{t=0}^{T} n_{ij}(t)}{\sum_{t=0}^{T}\left(\sum_{j=0}^{k} n_{ij}(t)\right)}, \tag{2.5}$$

where $\hat{p}_{ij}$ is the estimated parameter of the transition process (i.e., the homogeneous transition probability from grade i to grade j). Let $\mathbf{P} = [\hat{p}_{ij}]_{i,j \in S}$ be a $k \times k$ transition matrix between the states of the system and let $\mathbf{w}$ be a $k \times 1$ vector of loss probabilities, $\hat{p}_{i0}, i \in S$.

Since an individual in the system must either remain in the same grade, move to another grade, or leave the system, then the loss probability vector is expressed in terms of the transition matrix between the states as

$$\mathbf{w}' = (\mathbf{I} - \mathbf{P})\,\mathbf{e}', \tag{2.6}$$

where $\mathbf{e}'$ is a $k \times 1$ vector of ones and $\mathbf{I}$ is a $k \times k$ identity matrix. In matrix notation, the total stock of the system at time t is $N(t) = \mathbf{n}(t)\mathbf{e}'$. So the system expansion $\Delta N(t)$ can be expressed as (Woodward, 1983):

$$\Delta N(t) = \alpha \mathbf{n}(t)\mathbf{e}'. \tag{2.7}$$

Let $R(t+1)$ be the total number of recruits into the system during the period $(t, t+1)$. With the assumption that recruitment is done to replace wastage and to achieve the desired growth, the expected number of recruits, denoted by $E[R(t+1)]$, is expressed as

$$E[R(t+1)] = \mathbf{n}(t)\,(\mathbf{I} - \mathbf{P})\,\mathbf{e}' + \alpha \mathbf{n}(t)\mathbf{e}'. \tag{2.8}$$

The actual total number of recruits may vary from what is expected as a result of the external factors (3) and (4) mentioned in the Introduction. It may also be due to recruits who fail to resume for duty or the outsourcing of personnel with critical skills for specialized jobs after recruitment has been done in line with the assumed recruitment plan. To accommodate this situation and other unexplained variation in Eq. (2.8), we introduce the error term $\epsilon(t+1)$ so that Eq. (2.8) becomes

$$R(t+1) = \mathbf{n}(t)\,(\mathbf{I} - \mathbf{P})\,\mathbf{e}' + \alpha \mathbf{n}(t)\mathbf{e}' + \epsilon(t+1), \qquad E[\epsilon(t+1)] = 0. \tag{2.9}$$

Suppose there are available data at $t = 0, 1, 2, \ldots, T$, and the total recruits, $R(t+1)$, is homoscedastic, say $var[R(t+1)] = \sigma^2$. Then Eq. (2.9) can be expressed in a matrix-vector form as

$$R = \mathbf{A}\,(\mathbf{I} - \mathbf{P})\,\mathbf{e}' + \alpha N + \epsilon,$$

where

$$R = \begin{bmatrix} R(1) \\ R(2) \\ \vdots \\ R(T+1) \end{bmatrix}, \qquad \mathbf{A} = \begin{bmatrix} n_1(0) & n_2(0) & \cdots & n_k(0) \\ n_1(1) & n_2(1) & \cdots & n_k(1) \\ \vdots & \vdots & \ddots & \vdots \\ n_1(T) & n_2(T) & \cdots & n_k(T) \end{bmatrix}, \qquad N = \begin{bmatrix} N(0) \\ N(1) \\ \vdots \\ N(T) \end{bmatrix}, \qquad \text{and}$$

$\epsilon = \begin{bmatrix} \epsilon(1) \\ \epsilon(2) \\ \vdots \\ \epsilon(T+1) \end{bmatrix}$. We obtain the growth rate index by the method of least squares (i.e., minimizing the sum of squared errors, $\epsilon'\epsilon$, with respect to α). We do this because α is unknown to the researcher. Taking derivative with respect to α and setting the derivative to zero, we have

$$\tilde{\alpha} = \left(R' - \mathbf{e}\,(\mathbf{I} - \mathbf{P})'\,\mathbf{A}'\right) N (N'N)^{-1}, \tag{2.10}$$

where $\tilde{\alpha}$ is the new stable growth rate index. This stable growth rate index $\tilde{\alpha}$ is an unbiased estimator of α with variance $(N'N)^{-1}\sigma^2$. This is easy to see as

$$E[\tilde{\alpha}] = \left(E[R'] - \mathbf{e}\,(\mathbf{I}-\mathbf{P})'\,\mathbf{A}'\right) N(N'N)^{-1} = \alpha$$

and

$$var[\tilde{\alpha}] = N'(E[RR'] - \mathbf{A}\,(\mathbf{I}-\mathbf{P})\,\mathbf{e}'\mathbf{e}\,(\mathbf{I}-\mathbf{P})'\,\mathbf{A}' - 2\alpha N\mathbf{e}\,(\mathbf{I}-\mathbf{P})'\,\mathbf{A}' \\ -\alpha^2 NN')N(N'N)^{-2} = N'var[R]N(N'N)^{-2}.$$

In algebraic form Eq. (2.10) is written as

$$\tilde{\alpha} = \frac{\sum_{t=0}^{T}\left(\sum_{i=1}^{k} n_i(t)\left[R(t+1) - \sum_{i=1}^{k} n_i(t)\left(1-\sum_{i=1}^{k}\hat{p}_{ij}\right)\right]\right)}{\sum_{t=0}^{T}\left(\sum_{i=1}^{k} n_i(t)\right)^2}. \tag{2.11}$$

We can see that Eq. (2.11) is quite different from Eq. (2.1) and Eq. (2.2) in that the stocks, $n_i(t)$, the flows as captured by the transition probabilities, $\hat{p}_{ij}$, and the total recruits, $R(t+1)$, in the organizational system are accounted for in Eq. (2.11). The growth rate $\tilde{\alpha}$ is negative when $R(t+1) < \sum_{i=1}^{k} n_i(t)\left(1-\sum_{i=1}^{k}\hat{p}_{ij}\right)$. This is an indication that the organization is contracting over time (e.g., by forcing layoffs).

Relying on the Individual-Chart-Moving-Range (ICMR) within the context of statistical process control (Saniga, 1989), we create control limits for the manpower stocks generated by the growth rate. We adjudge whether the resulting stocks are under control; and if they are, the growth rate associated with them is said to be acceptable. To obtain the control interval, we treat each entry in the sequence of stock vectors, $\{\mathbf{n}(t)\}$, as a single subgroup. By taking the average of the entries, $\overline{n}_i$, over time and the average moving range, $\overline{MR}_i$, we get

$$\overline{n}_i = \frac{\sum_{t=0}^{T} n_i(t)}{T+1} \text{ and } \overline{MR}_i = \frac{\sum_{t=0}^{T}|\Delta n_i(t+1)|}{T}, \text{ respectively.}$$

The control interval is expressed as

$$CL_i = \left[\overline{n}_i - E_{2(k)}\overline{MR}_i\ ,\ \overline{n}_i + E_{2(k)}\overline{MR}_i\right], \tag{2.12}$$

where $E_{2(k)}$ is obtained from the tables of constants for control charts at a subgroup size of k. There is no need to stress the development of this control interval as it is straight forward from statistical process control (Saniga, 1989). The tables of constants for the control charts can be found on the website:

www.buec.udel.edu/kherh/table_of_control_charts_constants.pdf.

Since a process is said to be under control if it lies within the control interval, we use this idea as a conceptual underpinning to propose that a stable growth index, $1+\alpha^*$, is acceptable when $(1+\alpha^*)\overline{n}_i \in CL_i$ for all $i \in S$.

The stable growth index, $\langle\vartheta\rangle = 1+\tilde{\alpha}$, developed in this paper simplifies to

$$0 < \langle\vartheta\rangle = \frac{\sum_{t=0}^{T}\left(\sum_{i=1}^{k} n_i(t)\left[R(t+1) + \sum_{i=1}^{k} n_i(t)\sum_{i=1}^{k}\hat{p}_{ij}\right]\right)}{\sum_{t=0}^{T}\left(\sum_{i=1}^{k} n_i(t)\right)^2}. \tag{2.13}$$

It is important to mention here that the basis for referring to $\langle \vartheta \rangle$ as being stable follows from the desired growth rate of a stable manpower system.

2.3 *The use of $\langle \vartheta \rangle$ as an employment index*

We now illustrate the use of the stable growth index as an employment index. We consider the following cases.

Case I: Organizations on a zero recruitment policy, i.e., $R(t+1) = 0$ for all t. Since the operational policy of organizations is assumed to persist into the future, then

$$0 < \lim_{T\to\infty} \langle \vartheta \rangle = \frac{\sum_{t=0}^{\infty}\left(\sum_{i=1}^{k} n_i(t)\left[\sum_{i=1}^{k} n_i(t)\sum_{i=1}^{k}\hat{p}_{ij}\right]\right)}{\sum_{t=0}^{\infty}\left(\sum_{i=1}^{k} n_i(t)\right)^2}.$$

If losses do not occur, then $\mathbf{P}$ is stochastic, i.e. $\sum_{i=1}^{k}\hat{p}_{ij} = 1$, and $\lim_{T\to\infty}\langle \vartheta \rangle = 1$. Conversely, if losses do occur, then $\mathbf{P}$ is sub-stochastic, i.e. $\sum_{i=1}^{k}\hat{p}_{ij} < 1$, and $0 < \lim_{T\to\infty}\langle \vartheta \rangle < 1$.

Case II: An organization where employment is just to replace wastage. Here,

$$R(t+1) = \sum_{i=1}^{k} n_i(t)\left(1 - \sum_{i=1}^{k}\hat{p}_{ij}\right).$$

Thus $\lim_{T\to\infty}\langle \vartheta \rangle = 1$.

Notice that, in either of the first two cases, $\sum_{i=1}^{k}\hat{p}_{ij} \le 1$ and $0 < \lim_{T\to\infty}\langle \vartheta \rangle \le 1$.

Case III: An organization on the vanguard for employment. In this case, $R' > \mathbf{e}\left(\mathbf{I} - \mathbf{P}\right)'\mathbf{A}'$. Equivalently, we write

$$R(t+1) > \sum_{i=1}^{k} n_i(t)\left(1 - \sum_{i=1}^{k}\hat{p}_{ij}\right).$$

After some algebra, we get

$$\frac{\sum_{t=0}^{T}\left(\sum_{i=1}^{k} n_i(t)\left[R(t+1) + \sum_{i=1}^{k} n_i(t)\sum_{i=1}^{k}\hat{p}_{ij}\right]\right)}{\sum_{t=0}^{T}\left(\sum_{i=1}^{k} n_i(t)\right)^2} > 1.$$

Taking limit as $T \to \infty$, we have $\lim_{T\to\infty}\langle \vartheta \rangle > 1$.

Thus $\langle \vartheta \rangle > 1$ is an indication that an organization is on the vanguard for employment, while $0 < \langle \vartheta \rangle \le 1$ signifies an organization clogging up on employment for sufficiently large t. The index $\langle \vartheta \rangle$ can be used to compare between organizations so as to identify the one that is doing more on employment creation. Suppose there are two organizations, A and B. Then A is adjudged to contribute more to employment than B when $\langle \vartheta \rangle_A > \langle \vartheta \rangle_B \ge 1$.

3. Illustrative example

We demonstrate the utility of the new growth index using data on academic staff flows for the Faculty of Physical Sciences in the University of Benin, Nigeria. The grades of academic staff in the faculty encompass – Graduate Assistant, Assistant Lecturer, Lecturer II, Lecturer I, Senior Lecturer, Associate Professor and Professor. These grades form the transient states of the university manpower system. The grades of academic staff in ascending order of ranks are represented in a set S as $S = \{1, 2, \cdots, 7\}$. This system also contains an absorbing state. The absorbing state encompasses all forms of wastage (or losses) such as retirement, resignation, death, etc. We collate data from the faculty's prospectus, from 2005/2006 session through 2011/2012 session. The data are displayed in a matrix-vector form as shown below:

$$\mathbf{F}_{ii} = \begin{bmatrix} 2 & 3 & 6 & 17 & 18 & 5 & 17 \\ 2 & 10 & 16 & 23 & 23 & 2 & 15 \\ 3 & 12 & 14 & 23 & 26 & 3 & 13 \\ 7 & 7 & 11 & 17 & 20 & 1 & 13 \\ 9 & 1 & 6 & 9 & 19 & 8 & 14 \\ 12 & 7 & 11 & 22 & 26 & 7 & 14 \end{bmatrix}, \qquad \mathbf{F}_{i(i+1)} = \begin{bmatrix} 5 & 8 & 10 & 6 & 0 & 1 \\ 1 & 0 & 1 & 3 & 1 & 2 \\ 1 & 0 & 2 & 0 & 0 & 0 \\ 0 & 8 & 3 & 8 & 7 & 2 \\ 3 & 6 & 12 & 11 & 8 & 0 \\ 3 & 0 & 1 & 0 & 3 & 9 \end{bmatrix},$$

$$\mathbf{F}_{i0} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 3 \\ 0 & 2 & 0 & 1 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \qquad R = \begin{bmatrix} 6 \\ 5 \\ 9 \\ 5 \\ 11 \\ 26 \end{bmatrix},$$

$$\mathbf{A} = \begin{bmatrix} 8 & 11 & 16 & 23 & 19 & 6 & 18 \\ 3 & 10 & 17 & 27 & 24 & 5 & 18 \\ 4 & 14 & 16 & 24 & 26 & 3 & 17 \\ 7 & 15 & 14 & 25 & 27 & 3 & 14 \\ 11 & 8 & 19 & 20 & 28 & 8 & 15 \\ 15 & 7 & 12 & 22 & 30 & 16 & 14 \end{bmatrix}, \qquad N = \begin{bmatrix} 101 \\ 104 \\ 104 \\ 105 \\ 109 \\ 116 \end{bmatrix},$$

where $\mathbf{F}_{ij}$ is the matrix of flows between grade i and j over the six consecutive sessions. Each row entry in the matrices and vectors is an instant of time. The column entries for the $\mathbf{F}_{ij}$'s and $\mathbf{A}$ represent the states of the system.

Using equation (2.5) we estimate the transition probabilities between the states, $i, j \in S$, of the system as:

$$\begin{aligned} &\hat{p}_{11} = 0.7083, \hat{p}_{12} = 0.2708, \\ &\qquad \hat{p}_{22} = 0.6154, \hat{p}_{23} = 0.3385, \\ &\qquad\qquad \hat{p}_{33} = 0.6809, \hat{p}_{34} = 0.3085, \\ &\qquad\qquad\qquad \hat{p}_{44} = 0.7872, \hat{p}_{45} = 0.1986, \\ &\qquad\qquad\qquad\qquad \hat{p}_{55} = 0.8571, \end{aligned}$$

$\hat{p}_{56} = 0.1234, \hat{p}_{66} = 0.6341, \hat{p}_{67} = 0.3415$, $\hat{p}_{77} = 0.8958$, $\hat{p}_{ji} = 0$, for $j > i$, and $\hat{p}_{ij} = 0$, for $i > j = i+1$. We carry out all computations in the MATLAB environment (see Appendix). The transition probability estimates, $\hat{p}_{ij}$, show that the flow pattern in the system follows a natural order (i.e., from one state to the next higher state). More so, the row sums of the transition probability estimates are less than one (i.e., the transition matrix is sub-stochastic). This is because losses do occur in the system. By means of the information obtained from the prospectus and the obtained transition probability estimates, the stable growth index, $\tilde{\alpha}$, is found to be 0.0670.

Using the stable growth rate indices in Eq. (2.1) and Eq. (2.2), we obtain $\hat{\alpha} = 0.0244$ and $\bar{\hat{\alpha}} = 0.0283$, respectively. The calculated ICMR control intervals for each state using Eq. (2.12) are given in Table 1. These control intervals give some idea of how near the mean and the observed stocks is likely to fall. The average state-wise stocks as determined by each stable growth index are also obtained. It can be seen from Table 1 that the three

Table 1. ICMR control intervals and the average state-wise stocks

i	Control Intervals	$\overline{n}_i$	$(1+\hat{\alpha})\overline{n}_i$	$(1+\bar{\hat{\alpha}})\overline{n}_i$	$(1+\tilde{\alpha})\overline{n}_i$
1	[4.2294 , 11.7706]	8	8.1950	8.2266	8.5357
2	[7.7281 , 13.9385]	10.8333	11.0974	11.1402	11.5587
3	[12.1179 , 19.2155]	15.6667	16.0485	16.1105	16.7157
4	[20.1730 , 26.8270]	23.5000	24.0727	24.1657	25.0735
5	[23.2269 , 28.1065]	25.6667	26.2922	26.3937	27.3853
6	[3.2845 , 10.3821]	6.8333	6.9999	7.0269	7.2909
7	[14.6692 , 17.3308]	16	16.3900	16.4532	17.0713

growth indices give average state-wise stocks which are in control. The results suggest that, particularly the manpower of the faculty is an expanding one, the estimated growth rate indices are acceptable. Thus the growth index, $1 + \tilde{\alpha}$, is a viable alternative to the existing ones in the literature. Without loss of generality, $\langle \vartheta \rangle = 1.0670$ is appropriate within the assumptions of the model as indicated by the ICMR. The value 1.0670 of $\langle \vartheta \rangle$ indicates that, although the transition matrix is sub-stochastic and losses do occur in the system, the recruitment level is enough to expand the total stock of the system. Hence the university-faculty can be adjudged to have been on the vanguard for employment over time.

4. Conclusion

In this study, we have developed a stable growth index that enables us have an insight into whether or not an organization is on the vanguard for employment. The stable growth index settles the concerns and future research direction of McClean (1976a) on the need to build a growth index that incorporates the gradewise structure and promotion rates. The stable growth index accounts for the recruitment flows, the transition probabilities and the stocks in the system. The index serves as a viable alternative to the existing growth factor estimators in the literature. The main advantage of our method is that it not only provides an estimate for an organization growth but can also be used as a basis to adjudge whether the organization is on the vanguard for employment. We provide three case scenarios to show how the stable groth index, $\langle \vartheta \rangle$, can be used to investigate whether an organization is on the vanguard for employment or whether it is clogging up on it. In this regard, the new stable growth index was applied to a university-faculty manpower structure. The results show that the manpower stocks expand over time within an acceptable ICMR control interval, which means that the university-faculty was on the vanguard for employment. However, it may be difficult to ascertain whether most public corporations and even multinational companies in developing countries are on the vanguard for employment as we do not directly have access to their manpower data bank.

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References

Bartholomew, D. J., Forbes, A. F., and McClean, S. I. (1991). *Statistical Techniques for Manpower Planning* (2nd ed.). John Wiley & Sons, Chichester.

Ekhosuehi, V. U. and Osagiede, A. A. (2007). An analytic study of unemployment scenario in Nigeria. *Journal of Social & Management Sciences*, 12: 84-91.

Elikwu, A. B. (2003). The relationship between unemployment and economic growth in Nigeria. *The Nigerian Economic and Financial Review*, 8(2): 65–81.

Even-Dar, E., Kakade, S. M. and Mansour, Y. (2009). Online Markov decision processes. *Mathematics of Operations Research*, 34(3): 726–736.

Hopkins, D. S. P. (1980). Models for affirmative action planning and evaluation. *Management Science*, Vol. 26(10): 994–1006.

Ibe, C. O. (2009). *Markov Processes for Stochastic Modelling.* Elsevier Academic Press, Burlington, USA.

Jhingan, M. L. (2003). *The Economics of Development and Planning* (36th Ed.). Vrianda Publications (P) Ltd, Delhi.

Kim, S. H. (1985). A stability analysis of growth matrix in co-operating enterprises. *Journal of the Operational Research Society*, 36(12): 1129–1131.

McClean, S. (1976a). The two-stage model of personnel behaviour. *Journal of the Royal Statistical Society. Series A (General)*, 139(2): 205–217.

McClean, S. (1976b). Some models for company growth. *Journal of the Royal Statistical Society. Series A (General)*, 139(4): 501–507.

Osagiede, A. A. and Ekhosuehi, V. U. (2006). Markovian approach to school enrolment projection process. *Global Journal of Mathematical Sciences*, 5(1): 1–7.

Osagiede, A. A., Ekhosuehi, V. U., and Osagiede, F. E. U. (2007). Avoiding staff retrenchment in manpower organisations. *Journal of the Mathematical Association of Nigeria* (abacus), 34 (2A): 107–112.

Osagiede, A. A. and Omosigho, S. E. (2004). School enrolment projection using spreadsheet under scanty data. *Nigerian Annals of Natural Sciences*, 5(2): 87–99.

Saniga, E. M. (1989). Economic statistical control-chart designs with an application to $\bar{X}$ and R charts. *Technometrics*, 31(3): 313–320.

Setlhare, K. (2007). Optimization and estimation study of manpower planning models. *PhD Thesis*, University of Pretoria, South Africa.

Vassiliou, P.-C. G. (1976). A Markov chain model for wastage in manpower systems. *Operations Research Quarterly* (1970–1977), 27(1, Part 1): 57–70.

Vassiliou, P.-C. G. (1981). Stability in a non-homogeneous Markov chain model in manpower systems. *Journal of Applied Probability*, 18(4): 924–930.

Woodward, M. (1983). On forecasting grade, age and length of service distributions in manpower systems. *Journal of the Royal Statistical Society*, Series A (General), 146(1): 74–84.

Zanakis, S. H. and Maret, M. W. (1980). A Markov chain application to manpower supply planning. *Journal of the Operational Research Society*, 31: 1095–1102.

APPENDIX

MATLAB codes for computing the growth rate indices.

```
clc
% Data decleration
f11 =[2 2 3 7 8 12]; f12 =[5 1 1 0 3 3];
f10 =[1 0 0 0 0 0];
f22 =[3 10 12 7 1 7]; f23 =[8 0 0 8 6 0];
f20 =[0 0 2 0 1 0];
f33 =[6 16 14 11 6 11]; f34 =[10 1 2 3 12 1];
f30 =[0 0 0 0 1 0];
f44 =[17 23 23 17 9 22]; f45 =[6 3 0 8 11 0];
f40 =[0 1 1 0 0 0];
f55 =[18 23 26 20 19 26]; f56 =[0 1 0 7 8 3];
f50 =[1 0 0 0 1 1];
f66 =[5 2 3 1 8 7]; f67 =[1 2 0 2 0 9];
f60 =[0 1 0 0 0 0];
f77 =[17 15 13 13 14 14]; f70 =[1 3 4 1 1 0];

R =[6; 5; 9; 5; 11; 26];

fs1 =[8 3 4 7 11 15]; fs2 =[11 10 14 15 8 7];
fs3 =[16 17 16 14 19 12]; fs4 =[23 27 24 25 20 22];
fs5 =[19 24 26 27 28 30]; fs6 =[6 5 3 3 8 16];
fs7 =[18 18 17 14 15 14];

A=[fs1(1,1) fs2(1,1) fs3(1,1) fs4(1,1) fs5(1,1) fs6(1,1) fs7(1,1); ...
fs1(1,2) fs2(1,2) fs3(1,2) fs4(1,2) fs5(1,2) fs6(1,2) fs7(1,2);...
fs1(1,3) fs2(1,3) fs3(1,3) fs4(1,3) fs5(1,3) fs6(1,3) fs7(1,3); ...
fs1(1,4) fs2(1,4) fs3(1,4) fs4(1,4) fs5(1,4) fs6(1,4) fs7(1,4);...
fs1(1,5) fs2(1,5) fs3(1,5) fs4(1,5) fs5(1,5) fs6(1,5) fs7(1,5); ...
fs1(1,6) fs2(1,6) fs3(1,6) fs4(1,6) fs5(1,6) fs6(1,6) fs7(1,6)];

NA=[sum([fs1(1,1) fs2(1,1) fs3(1,1) fs4(1,1) ...
fs5(1,1) fs6(1,1) fs7(1,1)]);...
sum([fs1(1,2) fs2(1,2) fs3(1,2) fs4(1,2) fs5(1,2) fs6(1,2) fs7(1,2)]);...
sum([fs1(1,3) fs2(1,3) fs3(1,3) fs4(1,3) fs5(1,3) fs6(1,3) fs7(1,3)]); ...
sum([fs1(1,4) fs2(1,4) fs3(1,4) fs4(1,4) fs5(1,4) fs6(1,4) fs7(1,4)]);...
sum([fs1(1,5) fs2(1,5) fs3(1,5) fs4(1,5) fs5(1,5) fs6(1,5) fs7(1,5)]); ...
sum([fs1(1,6) fs2(1,6) fs3(1,6) fs4(1,6) fs5(1,6) fs6(1,6) fs7(1,6)])];


disp('The proposed index')
pf11=sum(f11)/sum(fs1); pf12=sum(f12)/sum(fs1);
pf22=sum(f22)/sum(fs2); pf23=sum(f23)/sum(fs2);
```

```
pf33=sum(f33)/sum(fs3); pf34=sum(f34)/sum(fs3);
pf44=sum(f44)/sum(fs4); pf45=sum(f45)/sum(fs4);
pf55=sum(f55)/sum(fs5); pf56=sum(f56)/sum(fs5);
pf66=sum(f66)/sum(fs6); pf67=sum(f67)/sum(fs6);
pf77=sum(f77)/sum(fs7);

Pf=[pf11(1,1) pf12(1,1) 0 0 0 0 0; 0 pf22(1,1) pf23(1,1) 0 0 0 0; ...
0 0 pf33(1,1) pf34(1,1) 0 0 0; 0 0 0 pf44(1,1) pf45(1,1) 0 0; ...
0 0 0 0 pf55(1,1) pf56(1,1) 0; 0 0 0 0 0 pf66(1,1) pf67(1,1);...
0 0 0 0 0 0 pf77(1,1)],

s=7; e=ones(1,s);
alpha=(NA'*R-(e*(eye(s)-Pf)')*A'*NA)*inv(NA'*NA),

x1=[8 3 4 7 11 15];
x2=[11 10 14 15 8 7];
x3=[16 17 16 14 19 12];
x4=[23 27 24 25 20 22];
x5=[19 24 26 27 28 30];
x6=[6 5 3 3 8 16];
x7=[18 18 17 14 15 14];

N=[101 104 104 105 109 116];
t=1:6;
a0=exp((12*sum(t.*log(N))-6*(7)*sum(log(N)))/(6*35))-1,

a1=mean([(NA(2,1)-NA(1,1))/NA(1,1) (NA(3,1)-NA(2,1))/NA(2,1) ...
(NA(4,1)-NA(3,1))/NA(3,1) (NA(5,1)-NA(4,1))/NA(4,1) ...
(NA(6,1)-NA(5,1))/NA(5,1)]),

r1=[abs(x1(1,2)-x1(1,1)), abs(x1(1,3)-x1(1,2)), abs(x1(1,4)-x1(1,3)), ...
abs(x1(1,5)-x1(1,4)), abs(x1(1,6)-x1(1,5))],

r2=[abs(x2(1,2)-x2(1,1)), abs(x2(1,3)-x2(1,2)), abs(x2(1,4)-x2(1,3)), ...
abs(x2(1,5)-x2(1,4)), abs(x2(1,6)-x2(1,5))],

r3=[abs(x3(1,2)-x3(1,1)), abs(x3(1,3)-x3(1,2)), abs(x3(1,4)-x3(1,3)), ...
abs(x3(1,5)-x3(1,4)), abs(x3(1,6)-x3(1,5))],

r4=[abs(x4(1,2)-x4(1,1)), abs(x4(1,3)-x4(1,2)), abs(x4(1,4)-x4(1,3)), ...
abs(x4(1,5)-x4(1,4)), abs(x4(1,6)-x4(1,5))],

r5=[abs(x5(1,2)-x5(1,1)), abs(x5(1,3)-x5(1,2)), abs(x5(1,4)-x5(1,3)), ...
abs(x5(1,5)-x5(1,4)), abs(x5(1,6)-x5(1,5))],

r6=[abs(x6(1,2)-x6(1,1)), abs(x6(1,3)-x6(1,2)), abs(x6(1,4)-x6(1,3)), ...
abs(x6(1,5)-x6(1,4)), abs(x6(1,6)-x6(1,5))],

r7=[abs(x7(1,2)-x7(1,1)), abs(x7(1,3)-x7(1,2)), abs(x7(1,4)-x7(1,3)), ...
abs(x7(1,5)-x7(1,4)), abs(x7(1,6)-x7(1,5))],

n=length(x1),

MR1=sum(r1)/(n-1),
```

```
MR2=sum(r2)/(n-1),
MR3=sum(r3)/(n-1),
MR4=sum(r4)/(n-1),
MR5=sum(r5)/(n-1),
MR6=sum(r6)/(n-1),
MR7=sum(r7)/(n-1),

mux1=mean(x1), mux2=mean(x2), mux3=mean(x3), mux4=mean(x4),
mux5=mean(x5), mux6=mean(x6), mux7=mean(x7),

E2=1.109;

CLX1=[mux1-E2*MR1,mux1+E2*MR1], CLX2=[mux2-E2*MR2,mux2+E2*MR2],
CLX3=[mux3-E2*MR3,mux3+E2*MR3], CLX4=[mux4-E2*MR4,mux4+E2*MR4],
CLX5=[mux5-E2*MR5,mux5+E2*MR5], CLX6=[mux6-E2*MR6,mux6+E2*MR6],
CLX7=[mux7-E2*MR7,mux7+E2*MR7],

CX1=(1+alpha)*mux1,
CX2=(1+alpha)*mux2,
CX3=(1+alpha)*mux3,
CX4=(1+alpha)*mux4,
CX5=(1+alpha)*mux5,
CX6=(1+alpha)*mux6,
CX7=(1+alpha)*mux7,

CX1a0=(1+a0)*mux1,
CX2a0=(1+a0)*mux2,
CX3a0=(1+a0)*mux3,
CX4a0=(1+a0)*mux4,
CX5a0=(1+a0)*mux5,
CX6a0=(1+a0)*mux6,
CX7a0=(1+a0)*mux7,

CX1a1=(1+a1)*mux1,
CX2a1=(1+a1)*mux2,
CX3a1=(1+a1)*mux3,
CX4a1=(1+a1)*mux4,
CX5a1=(1+a1)*mux5,
CX6a1=(1+a1)*mux6,
CX7a1=(1+a1)*mux7,

vartheta=1+alpha,
```