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Exponential ratio-product type estimators in stratified random sampling

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In this study, exponential ratio-product type estimator of population mean in stratified random sampling without replacement is proposed. The expressions for bias and Mean Squared Error (MSE) are derived. Comparisons of the proposed estimator with conventional estimator among others for the entire strata showed that the proposed estimator is more efficient than the competing estimators as it possessed minimum variance and hence minimum coefficient of variations.

Keywords: exponential estimator; stratification; correlation; mean square error.

1. Introduction

To estimate population characteristics, survey statisticians are usually interested in estimators that are efficient and possess other desirable properties required for a good estimator. In situations where conventional estimators become inefficient, it is appropriate to develop an appropriate alternative estimator for estimating population characteristics. The exponential form of the conventional ratio and product estimators has been found to be suitable alternatives (Diana *et al.*, 2011, Solanki, *et al.* 2012). Yet, the study of these alternative estimators can be improved upon by employing the correlation coefficient between the study and auxiliary variables in each stratum so as to extend the application of exponential ratio-product type estimators in stratified sampling. Of course, the modified ratio-product exponential estimator of the finite population mean of a study variable using auxiliary information has great significance in various fields, such as agriculture, in planning, manufacturing industries or in quality control etc. (Yadav and Mishra, 2015).

There are several estimators existing in literature that utilizes auxiliary variables in developing alternative estimators as contained in the works of Kadilar and Cingi (2005, 2006), Singh *et al.* (2008), Tailor *et al.* (2016), Kungu and Odongo, (2014), Vishwakarma and Kumar (2015) among others. Notable contribution to this study is the work by Khoshevisian *et al.* (2007) who presented a general family of estimators of y under the SRSWOR scheme using known parameters of the auxiliary variable x namely; standard deviation, correlation, coefficient skewness, kurtosis and coefficient of variation, etc. Similarly, Singh and Tailor (2010) proposed two ratio and product type estimators using transformation based on known minimum and maximum values of auxiliary variables.

The idea of modified estimators as Rashid *et al.* (2015) observed is that the exponential estimators are preferable to classical ratio and product estimators when the linear relationship between study and auxiliary variable is not very strong.

Sanaullah *et al.* (2012) presented an improved exponential ratio type estimator in survey sampling. Some developments included work by Riaz *et al.* (2014) that developed an estimator by combining the concept of Bahl and Tuteja (1991) exponential type estimator and classical regression estimator. Yadav and Adewara, (2013) worked on the estimation of population mean of the variable of study utilizing improved ratio-product type exponential estimator and qualitative auxiliary information and establish that the proposed estimator

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which under optimum conditions performs better than the usual sample mean estimator.

Other research papers on modified ratio-type, exponential ratio-type exponential product type and regression exponential type estimators based on different types of transformations have been carried out. Some important contributions in this area are due to Ozgul and Cingi (2014), Saini and Kumar (2015), Solanki *et al.* (2012), Sharma *et al.* (2013), Singh and Pal (2015), etc.

Stratified random sampling has often proved needful in improving the precision of estimates over simple random sampling. Many authors such as Singh, *et al.* (2009), Tailor, *et al.* (2013) developed various estimators to improve the ratio and Product estimators in Simple Random sampling while Singh and Kumar, (2012) suggested a generalized exponential estimator for two auxiliary variables in stratified sampling following, Singh *et al.* (2009).

Much literature has been produced on sampling from finite population to address the issue of the efficient estimation of the mean (or total) of a variable when auxiliary variables are available. This research work will therefore, construct hybrid type exponential ratio-product estimator in stratified random sampling considering various levels of correlation that exist amongst the variables in the each stratum.

2. Preliminaries and notations

Consider a finite population $U = (u_1, u_2, ..., u_N)$ of size N and it is divided into L strata of size $N_h (h = 1, 2, ..., L)$. Let Y be the Study variate and X be auxiliary variate taking value y_{hi} and x_{hi}, $h = 1, 2, ...L$, $i = 1, 2, ..., N_h$ on i^{th} unit of the h^{th} stratum which constitute a sample of size $n = \sum_{h=1}^{l} n_h$ and N: Population size, L: Number of strata in the population, N_h: Number of units in the stratum h, $h = 1, 2, ..., L$

$\overline{Y_h} = \frac{1}{N_h} \sum_{h=1}^{N_h} y_{hi}$: h^{th} Stratum mean for the study variate Y

$\overline{X}_h = \frac{1}{N_h} \sum_{h=1}^{N_h} x_{hi}$: h^{th} Stratum mean for the auxiliary variate X

$\overline{Y} = \frac{1}{N} \sum_{h=1}^{l} \sum_{i=1}^{N_h} y_{hi} = \frac{1}{N_h} \sum_{h=1}^{l} N_h \overline{Y_h} = \sum_{h=1}^{l} W_h \overline{Y_h} = \overline{Y}_{st}$: Population mean of the study variate Y

$\overline{X} = \frac{1}{N} \sum_{h=1}^{l} \sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{l} W_h \overline{X}_h = \overline{X}_{st}$: Population mean of the auxiliary variate X

$\overline{y}_h = \frac{1}{n_h} \sum_{h=1}^{n_h} y_{hi}$: Sample mean of the study variate Y for h^{th} stratum,

$\overline{x}_h = \frac{1}{n_h} \sum_{h=1}^{n_h} x_{hi}$: Sample mean of the auxiliary variate X for h^{th} stratum,

$W_h = \frac{N_h}{N}$: Stratum weight of h^{th} stratum

$R_h = \frac{Y_h}{X_h}$: Ratio of the population means for the h^{th} stratum

$S^2_{Yh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (Y_{hi} - \overline{Y}_h)^2$: Population variance of characteristics under study for the h^{th} stratum.

$S^2_{Xh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (X_{hi} - \overline{X}_h)^2$: Population variance of auxiliary characteristics for the h^{th} stratum.

$S_{XYh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (X_{hi} - \overline{X}_h)(Y_i - \overline{Y})$: Covariance between auxiliary and study characteristics for h^{th} stratum.

$C_{Yh} = \frac{S_{Yh}}{\overline{Y}_h}$: Coefficient of variation of characteristics under study for h^{th} stratum.

$C_{Xh} = \frac{S_{Xh}}{\overline{X}_h}$: Coefficient of variation of auxiliary characteristics for h^{th} stratum.

$\rho_h = \frac{S_{XY}}{S_X S_Y}$: Correlation coefficient between the auxiliary and the study variables for h^{th} stratum.

Furthermore, let $C_h = \rho_h (\frac{C_{yh}}{C_{xh}})$, $f_h = \frac{n_h}{N_h}$ be the h^{th} stratum sampling fraction and $\theta_h = \frac{1 - f_h}{n_h}$. Usually unbiased estimators of population means $\overline{Y}$ and $\overline{X}$ in stratified random sampling are defined respectively as $\overline{y}_{st} = \sum_{h=1}^{L} W_h \overline{y_h}$ and $\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_h$.

2.1 Conventional ratio and product estimators in stratified random sampling

The combined ratio estimator of population mean when $\overline{X}$ of auxiliary variate X is known is defined as

$$\overline{y}_{R,st} = \overline{y}_{st}\left(\frac{\overline{X}}{\overline{x}_{st}}\right), \overline{x} \neq 0, \overline{X} \neq 0 \tag{1}$$

It is assumed that the study variate and the auxiliary variate are positively correlated. When the study variate Y and the auxiliary variate X are negatively correlated, assuming that the population mean $\overline{X}$ of the auxiliary variate X is known, then the combined product estimator is defined as

$$\overline{y}_{P,st} = \overline{y}_{st}\left(\frac{\overline{x}_{st}}{\overline{X}}\right), \overline{x} \neq 0, \overline{X} \neq 0 \tag{2}$$

The biases of the estimators $\overline{y}_{R,st}$, and $\overline{y}_{P,st}$ up to the first order of approximations respectively are given as

$$Bias\left(\overline{y}_{R,st}\right) = \overline{Y_{st}}\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2(1-C_h) \ \ and \ \ Bias\left(\overline{y}_{P,st}\right) = \overline{Y_{st}}\sum_{h=1}^{L}\theta_h W_h^2 C_h C_{xh}^2 \tag{3}$$

while the MSEs of the conventional Ratio and Product estimators under stratified random sampling denoted as $MSE\left(\overline{y}_{R,st}\right)$ and $MSE\left(\overline{y}_{P,st}\right)$ are respectively given as

$$MSE\left(\overline{y}_{R,st}\right) = \overline{Y_{st}^2}\sum_{h=1}^{L} W_h^2\theta_h\left(C_{yh}^2 + C_{xh}^2(1-2C_h)\right) \tag{4}$$

and

$$MSE\left(\overline{y}_{P,st}\right) = \overline{Y_{st}^2}\sum_{h=1}^{L} W_h^2\theta_h\left(C_{yh}^2 + C_{xh}^2(1+2C_h)\right) \tag{5}$$

2.2 Exponential ratio-type and product-type estimators in stratified random sampling

The Bahl and Tuteja (1991) ratio and product type exponential estimators of population mean Y under stratified random sampling is given by:

$$\overline{y}_{Re,st} = \overline{y}_{st}\exp\left(\frac{\overline{X_{st}} - \overline{x_{st}}}{\overline{X_{st}} + \overline{x_{st}}}\right) \tag{6}$$

while the Product Exponential estimator in stratified random sampling is given by

$$\overline{y}_{Pe,st} = \overline{y}_{st}\exp\left(\frac{\overline{x_{st}} - \overline{X_{st}}}{\overline{x_{st}} + \overline{X_{st}}}\right) \tag{7}$$

The biases of the modified ratio and product exponential estimators under stratified random sampling are given as:

$$Bias\left(\overline{y}_{Re,st}\right) = \frac{\overline{Y_{st}}}{8}\sum_{h=1}^{L} W_h^2\theta_h C_{xh}^2(3-4C_h) \tag{8}$$

and

$$Bias\left(\overline{y}_{Pe,st}\right) = \frac{\overline{Y_{st}}}{8}\sum_{h=1}^{L} W_h^2\theta_h C_{xh}^2(4C_h-1) \tag{9}$$

The MSE of ratio exponential estimator $\overline{y}_{Re,st}$ and product exponential estimator $\overline{y}_{Pe,st}$ under stratified random sampling are given by

$$MSE\left(\overline{y}_{Re,st}\right) = \overline{Y_{st}^2}\sum_{h=1}^{L}\theta_h W_h^2\left(C_{yh}^2 + \frac{C_{xh}^2}{4}(1-4C_h)\right) \tag{10}$$

and

$$MSE\left(\overline{y}_{Pe,st}\right) = \overline{Y_{st}^2}\sum_{h=1}^{L}\theta_h W_h^2\left(C_{yh}^2 + \frac{C_{xh}^2}{4}(1+4C_h)\right) \tag{11}$$

2.3 ***Proposed ratio-product exponential estimator in stratified random sampling***

The Proposed ratio-product exponential estimator of in stratified random sampling is given by

$$\overline{y}_{RcP,st}^{e} = \overline{y_{st}}\left(\rho_h \exp\left(\frac{\overline{X_{st}}-\overline{x_{st}}}{\overline{X_{st}}+\overline{x_{st}}}\right) + (1-\rho_h)\exp\left(\frac{\overline{x_{st}}-\overline{X_{st}}}{\overline{x_{st}}+\overline{X_{st}}}\right)\right) \tag{12}$$

where ρ_h is the population coefficient between Y and X in the h^{th} stratum.

THEOREM 2.1 *The bias of alternative ratio-product exponential type estimator of finite population mean in stratified random sampling is given by:*

$$B\left(\overline{y}_{RcP,st}^{e}\right) = \overline{Y}_{st} = \sum_{h=1}^{L} W_h^2\theta_h C_{xh}^2\left(\frac{C_h}{2} - \rho_h^2 C_h - \frac{1}{8} + \frac{\rho_h}{2}\right) \tag{13}$$

THEOREM 2.2 *The MSE of improved ratio-product exponential estimator* $MSE\left(\overline{y}_{RcP,st}^{e}\right)$ *in stratified random sampling is given by:*

$$MSE\left(\overline{y}_{RcP,st}^{e}\right) = \overline{Y}_{st}^2 = \sum_{h=1}^{L}\theta_h W_h^2\left(C_{yh}^2 + C_{xh}^2\left(\frac{1}{4} - \rho_h + \rho_h^2 + C_h - 2\rho_h C_h\right)\right) \tag{14}$$

Proof. The proofs for Theorem 2.1 and Theorem 2.2 are found in Appendix 1 and Appendix 2 respectively. ■

Relationship between proposed estimator and other exponential Ratio-Product type estimator.

i. when $\rho_h = 0$, the exponential product type estimator is realized.
ii. when $\rho_h = 1$, the exponential ratio type estimator is realized;
iii. when $\rho_h = -1$, there realized estimator is a weighted average where the impact of the exponential product estimator is modified by the exponential ratio estimator.
iv. when $\rho_h = \pm k(k < 1)$, the two estimators provide a balancing weighted average

3. Comparison of estimators

Relevant conditions under which the proposed estimator $\overline{y}^e_{RcP,st}$ is better than the modified estimators of Bahl and Tuteja, (1991) and the conventional estimators are presented.

3.1 *Efficiency of the exponential estimators in stratified random sampling*

To compare the efficiency of the proposed estimator to other estimators, we compare the MSE of proposed estimator with MSEs of other estimators that are considered.

From (4), (5), (10), (11) and (14) of estimators $\overline{y}_{R,st}$, $\overline{y}_{P,st}$, $\overline{y}_{Re,st}$, $\overline{y}_{Pe,st}$ and $\overline{y}^e_{RcP,st}$, the proposed estimator $\overline{y}^e_{RcP,st}$ would be more efficient than the combined ratio estimator $\overline{y}_{R,st}$ if
$MSE(\overline{y}_{R,st}) - MSE(\overline{y}^e_{RcP,st}) > 0$, that is;

$$\Big(\frac{3}{4}\overline{Y}^2_{st}\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 - 3\overline{Y}^2_{st}\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 C_h + \overline{Y}^2_{st}\rho_h\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 \\ -\overline{Y}^2_{st}\rho_h^2\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 + 2\overline{Y}^2_{st}\rho_h\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 C_h\Big) > 0 \tag{15}$$

Similarly Proposed estimator $\overline{y}^e_{RcP,st}$ would be more efficient than combined product estimator $\overline{y}_{P,st}$ if $MSE(\overline{y}_{P,st}) - MSE(\overline{y}^e_{RcP,st}) > 0$, that is;

$$\Big(\frac{3}{4}\overline{Y}^2_{st}\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 + \overline{Y}^2_{st}\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 C_h + \overline{Y}^2_{st}\rho_h\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 \\ -\overline{Y}^2_{st}\rho_h^2\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 + 2\overline{Y}^2_{st}\rho_h\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 C_h\Big) > 0 \tag{16}$$

Again the proposed estimator $\overline{y}^e_{RcP,st}$ would be more efficient than the exponential ratio estimator $\overline{y}_{Re,st}$ if $MSE(\overline{y}_{Re,st}) - MSE(\overline{y}^e_{RcP,st}) > 0$

$$\Big(\overline{Y}^2_{st}\rho_h\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 - \overline{Y}^2_{st}\rho_h^2\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 - 2\overline{Y}^2_{st}\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 C_h \\ + \overline{2}Y^2_{st}\rho_h\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2 C_h\Big) > 0 \tag{17}$$

while proposed estimator $\overline{y}^e_{RcP,st}$ would be more efficient than the exponential product estimator $\overline{y}^e_{Pe,st}$ if $MSE(\overline{y}_{Pe,st}) - MSE(\overline{y}^e_{RcP,st}) > 0$, that is;

$$\left(\overline{Y}^2_{st}\rho_h \sum_{h=1}^{L} \theta_h W_h^2 C_{xh}^2 - \overline{Y}^2_{st}\rho_h^2 \sum_{h=1}^{L} \theta_h W_h^2 C_{xh}^2 + 2\overline{Y}^2_{st}\rho_h \sum_{h=1}^{L} \theta_h W_h^2 C_{xh}^2 C_h\right) > 0 \qquad (18)$$

Expressions (15) to (18) provide suitable conditions under which the suggested estimator $\overline{y}^e_{RcP,st}$ would have less MSE when compared with conventional ratio and product estimators $\overline{y}_{R,st}$ and $\overline{y}_{P,st}$ and modified ratio estimator $\overline{y}_{Re,st}$ and product estimator $\overline{y}_{Pe,st}$

4. Results

To examine the performance of the proposed estimator empirically we consider population data set with the variables Y= Household Expenditure, X = Household Income, $\rho = 0.902$. Further description of the population of study are shown on Table 1 relating to average household expenditure (Y) per month given the average monthly household income (X) in Agbadu-Makurdi, Nigeria. Here, $N = 244$, $N_1 = 183$, $N_2 = 50$ and $N_3 = 11$ with samples of sizes $n = 97$ $(n_1 = 73, n_2 = 20, n_3 = 4)$. The correlation coefficient of the study population is $\rho = 0.902$ (with stratum correlation coefficients $\rho_1 = 0.399$, $\rho_2 = 0.533$ and $\rho_3 = 0.0.725$ for respective stratum). Other summary statistics of the study population are shown in Appendix 3. The proposed estimator of population mean, $\overline{y}^e_{RP}$ is compared with the conventional Ratio ($\overline{y}_R$), product ($\overline{y}_P$), Ratio-Exponential ($\overline{y}_{Re}$) and Product-exponential ($\overline{y}_{Pe}$) estimators using efficiency criteria namely, Bias, MSE, SE, CV and RE (%) under simple random sampling and stratified random sampling.

Stratum 1 with, $\rho_1 = 0.399$ (weak positive correlation) shows that the proposed Ratio-Product exponential estimator ($\overline{y}^e_{Re,st}$) has minimum MSE of 2856265.12 followed by the Ratio-Exponential estimator ($\overline{y}_{Re,st}$) with MSE of 3177823.76. In stratum 2, $\rho_2 = 0.533$ which is moderate, and so, the Exponential-Ratio estimator ($\overline{y}_{Re,st}$) with $MSE(\hat{\overline{y}}_{Re,st}) = 215572.18$ is the most efficient estimator, followed by the proposed estimator with $MSE(\hat{\overline{y}}^e_{RP,st}) = 278431.17$. However, when $\rho_3 = 0.725$ which is fairly high, the conventional Ratio estimator satisfies the properties of minimum MSE. For the entire strata, the proposed estimates for the respective estimators are shown on Table 2.

Table 1. **Estimates of Population Characteristics for Each Stratum**

Stratum	Estimators	Mean	Bias	MSE
1	$\overline{y}_{R,st}$	32846.93	193.27	9932989.99
$r_1 = 0.399$	$\overline{y}_{P,st}$	41763.83	41.51	18267347.9
	$\overline{y}_{Re,st}$	34882.08	67.29	**3177823.76** **
	$\overline{y}_{Pe,st}$	39327.16	-8.59	7345002.37
	$\hat{\overline{y}}^e_{RP,st}$	3659.79	31.64	**2856265.12***
2	$\overline{y}_{R,st}$	20813.18	3.31	410478.9
$r_2 = 0.533$	$\overline{y}_{P,st}$	2067.28	2.12	1257786.28
	$\overline{y}_{Re,st}$	20779.63	0.98	**215572.18***
	$\overline{y}_{Pe,st}$	20714.19	0.38	639225.86
	$\hat{\overline{y}}^e_{RP,st}$	20749.67	1.23	**278431.17****
3	$\overline{y}_{R,st}$	8222.83	-1.38	**962760.90***
$r_3 = 0.725$	$\overline{y}_{P,st}$	7793.43	3.39	3108056.28
	$\overline{y}_{Re,st}$	8113.3	-0.94	**1260109.65****
	$\overline{y}_{Pe,st}$	7893.37	1.44	2332757.34
	$\hat{\overline{y}}^e_{RP,st}$	8052.82	0.39	1419150.22

Table 2. **Estimates of Mean, Bias, MSE, SE, CV, and SE for the Stratified Population**

Estimators	Mean	Bias	MSE	SE	CV	RE
$\hat{\bar{y}}_{R,st}$	61882.94	195.21	11306229.90	3362.47	5.4 $*$	100.0
$\hat{\bar{y}}_{P,st}$	70236.54	47.02	122633189.78	4757.43	6.8	141.49
$\hat{\bar{y}}_{Re,st}$	63773.01	67.32	4653505.58	2157.20	3.4 $**$	64.16
$\hat{\bar{y}}_{Pe,st}$	67934.68	-6.77	10316985.57	3212.01	4.7 $***$	95.53
$\hat{\bar{y}}^{e}_{RP,st}$	86130.72	33.26	**4626256.50**	**2150.87**	**2.5** $*$	63.97

$*$ implies the most efficient estimator, $**$ indicates second most efficient estimator, while $***$ the third most efficient estimator in the class of competing estimators.

In this case, the standard error of the proposed estimator $SE(\hat{\bar{y}}^{e}_{RP,st})$ is less than other estimators, followed by $SE\hat{\bar{y}}_{Re,st}$ with coefficient of variation as 2.5% and 3.4% respectively. This result shows that the proposed estimator is more efficient than that of Bahl and Tuteja (1991) exponential Ratio estimator and the conventional Ratio estimator.

5. Concluding remarks

The study proposed an exponential Ratio-Product estimator in stratified random sampling using the idea of Bahl and Tuteja (1991). It exploited the correlation that existed between study and auxiliary variables. Findings of this study showed that the suggested estimator is expected to perform relatively better than the existing and even the Bahl and Tuteja (1991) estimators when investigated in stratified random sampling considering various levels of correlation coefficients. Further studies may consider different samples sizes and correlations between study and auxiliary variables in each stratum to investigate relative efficiency of the exponential product-ratio estimator in stratified random sampling.

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Appendix 1: Proof to Theorem 2.1

For $\overline{y}^e_{RcP} = \overline{y}_{st}\left(\rho_h \exp\left(\frac{\overline{X}_{st}-\overline{x}_{st}}{\overline{X}_{st}+\overline{x}_{st}}\right) + (1-\rho_h)\exp\left(\frac{\overline{x}_{st}-\overline{X}_{st}}{\overline{x}_{st}+\overline{X}_{st}}\right)\right)$
we write $e_{0h} = \frac{\sum_{h=1}^{L} W_h\overline{Y}_h}{\overline{Y}}$, $e_{1h} = \frac{\sum_{h=1}^{L} W_h\overline{X}_h}{\overline{X}}$, such that $\bar{y}_h = \bar{Y}_h(1+e_{0h})$, $\bar{x}_h = \bar{X}_h(1+e_{1h})$. Under stratified random sampling without replacement, we write $E(e_{0h}) = E(e_{1h}) = 0$, and $E(e_{0h}^2) = \theta_h(C_{yh}^2$, $E(e_{0h}e_{1h}) = \theta_h\rho_h C_{xh}C_{yh}$ where θ_h, C_{yh}^2,C_{xh}^2,ρ_h, C_{xh} and C_{yh} are defined in 2.
Now, $\overline{y}^e_{RcP,st} = \overline{y}_{st}\left[\rho_h \exp\left(\frac{\overline{X}_{st}-\overline{x}_{st}}{\overline{X}_{st}+\overline{x}_{st}}\right) + (1-\rho_h)\exp\left(\frac{\overline{x}_{st}-\overline{X}_{st}}{\overline{x}_{st}+\overline{X}_{st}}\right)\right]$

$$= \sum_{h=1}^{L} W_h\overline{Y}_h\big(1+e_{0h}\big)\left(\rho_h \exp\left(\frac{\sum_{h=1}^{L} W_h\overline{X}_h - \sum_{h=1}^{L} W_h\overline{X}_h\big(1+e_{1h}\big)}{\sum_{h=1}^{L} W_h\overline{X}_h - \sum_{h=1}^{L} W_h\overline{X}_h\big(1+e_{1h}\big)}\right)\right.$$
$$\left. +(1-\rho_h)\exp\left(\frac{\sum_{h=1}^{L} W_h\overline{X}_h\big(1+e_{1h}\big) - \sum_{h=1}^{L} W_h\overline{X}_h}{\sum_{h=1}^{L} W_h\overline{X}_h\big(1+e_{1h}\big) + \sum_{h=1}^{L} W_h\overline{X}_h}\right)\right)$$

$$= \sum_{h=1}^{L} W_h\overline{Y}_h\big(1+e_{0h}\big)\left(\rho_h \exp\left[\frac{-e_{1h}}{2+e_{1h}}\right] + (1-\rho_h)\exp\left[\frac{e_{1h}}{2+e_{1h}}\right]\right) \tag{12.1}$$

Let $u_h = \frac{-e_{1h}}{2+e_{1h}}$ and $v_h = \frac{e_{1h}}{2+e_{1h}}$ so that (12.1) becomes:
$\overline{y}^e_{RcP,st} = \sum_{h=1}^{L} W_h\overline{Y}_h\big(1+e_{0h}\big)\big(\rho_h e^{u_h} + (1-\rho_h)e^{v_h}\big)$

$$= \sum_{h=1}^{L} W_h\overline{Y}_h\big(1+e_{0h}\big)\left[\rho_h\left(1+u_h+\frac{u_h^2}{2!}+\cdots\right) + \big(1-\rho_h\big)\left(1+v_h+\frac{v_h^2}{2!}+\cdots\right)\right] \tag{12.2}$$

Substituting the values of u_h and v_h in (12.2) we get

$$\overline{y}^e_{RcP,st} = \sum_{h=1}^{L} W_h\overline{Y}_h\big(1+e_{0h}\big) = \left[\rho_h\left[1+\left(\frac{-e_{1h}}{2+e_{1h}}\right)+\left(\frac{-e_{1h}}{2(2+e_{1h})}\right)^2\right]\right.$$
$$\left. +(1-\rho_h)\left[1+\left(\frac{e_{1h}}{2+e_{1h}}\right)+\left(\frac{-e_{1h}}{2(2+e_{1h})}\right)^2\right]\right]$$

$$= \sum_{h=1}^{L} W_h \overline{Y}_h \left(1+e_{0h}\right)\left(\rho_h - \frac{\rho_h e_{1h}}{2+e_{1h}} + \frac{\rho_h e_{1h}^2}{\left(8+8e_{1h}+2e_{1h}^2\right)} + 1 + \frac{e_{1h}}{2+e_{1h}} + \frac{e_{1h}^2}{\left(8+8e_{1h}+2e_{1h}^2\right)} - \rho_h - \frac{\rho_h e_{1h}}{2+e_{1h}} - \frac{\rho_h e_{1h}^2}{\left(8+8e_{1h}+2e_{1h}^2\right)}\right)$$

$$= \sum_{h=1}^{L} W_h \overline{Y}_h \left(1+e_{0h}\right)\left(1 + \frac{e_{1h}}{2+e_1} - \frac{2\rho_h e_{1h}}{2+e_{1h}} + \frac{e_{1h}^2}{\left(8+8e_{1h}+2e_{1h}^2\right)}\right) \quad (12.3)$$

Expanding and substituting $= \sum_{h=1}^{L} W_h \overline{Y}_h$ from (12.3) we have:

$$\overline{y}^e_{RcP,st} = \overline{Y}_{st}\left(1+e_{0h}\right)\left(1 + \frac{e_{1h}}{2+e_{1h}} - \frac{2\rho_h e_{1h}}{2+e_{1h}} + \frac{e_{1h}^2}{\left(8+8e_{1h}+2e_{1h}^2\right)} + e_{0h} + \frac{e_{0h}e_{1h}}{2+e_{1h}} - \frac{2\rho_h e_{0h} e_{1h}}{2+e_{1h}}\right)$$

$$= \overline{Y}_{st} = \overline{Y}_{st}\left(e_{0h} + \frac{e_{1h}}{2+e_{1h}} - \frac{2\rho_h e_{1h}}{2+e_{1h}} + \frac{e_{0h}e_{1h}}{2+e_{1h}} - \frac{2\rho_h e_{0h} e_{1h}}{2+e_{1h}} - \frac{e_{1h}^2}{8+8e_{1h}+2e_{1h}^2}\right) \quad (12.4)$$

Subtracting from both side of (12.4) we have

$$\overline{y}^e_{RcP,st} - \overline{Y}_{st} = \overline{Y}_{st}\left(\frac{e_{0h}\left(8+8e_{1h}+2e_{1h}^2\right) + e_{1h}(4+2e_{1h}) - 2\rho_h e_{1h}(4+2e_{1h}) + e_{0h}e_{1h}(4+2e_{1h})}{(2+e_{1h})(4+2e_{1h})} \frac{-2\rho_h e_{0h} e_{1h}(4+2e_{1h}) + e_{1h}^2}{(2+e_{1h})(4+2e_{1h})}\right)$$

$$= \overline{Y}_{st}\left(\frac{8e_{0h} + 8e_{0h}e_{1h} + 4e_{1h} + 2e_{1h}^2 - 8\rho_h e_{1h} - 4\rho_h e_{1h}^2 + 4e_{0h}e_{1h} - 8\rho_h e_{0h} e_{1h} + e_{1h}^2}{2(2+e_{1h})^2}\right)$$

$$= \frac{\overline{Y}_{st}}{2}\left(\left(8e_{0h} + 4e_{1h} + 12e_{0h}e_{1h} - 8\rho_h e_{1h} - 8\rho_h e_{0h} e_{1h} + 3e_{1h}^2 - 4\rho_h e_{1h}^2\right)\left(2+e_{1h}\right)^{-2}\right)$$

$$= \frac{\overline{Y}_{st}}{2}\left(\left(8e_{0h} + 4e_{1h} + 12e_{0h}e_{1h} - 8\rho_h e_{1h} - 8\rho_h e_{0h} e_{1h} + 3e_{1h}^2 - 4\rho_h e_{1h}^2\right)\left(\frac{1}{4} - \frac{e_{1h}}{4}\right)\right) \quad (12.5)$$

Opening the brackets (12.5) we have:

$$\overline{y}^e_{RcP,st} - \overline{Y}_{st} = \frac{\overline{Y}_{st}}{2}\left(2e_{0h} - 2e_{0h}e_{1h} + e_{1h} - e_{1h}^2 + 3e_{0h}e_{1h} - 2\rho_h e_{1h} + 2\rho_h e_{1h}^2 - 2\rho_h e_{0h} e_{1h} + \frac{3e_{1h}^2}{4} - \rho_h e_{1h}^2\right)$$

$$= \frac{\overline{Y}_{st}}{2}\left(2e_{0h} + e_{1h} + e_{0h}e_{1h} - 2\rho_h e_{1h} - 2\rho_h e_{0h}e_{1h} - \frac{e_{1h}^2}{4} + \rho_h e_{1h}^2\right) \quad (12.6)$$

To obtain the bias of $\overline{y}^e_{RcP,st}$ up to the first order of approximation, we take expectation on both sides of (12.6), so that:

$$B\left(\overline{y}^e_{RcP,st}\right) = E\left(\overline{y}^e_{RcP,st} - \overline{Y}_{st}\right)$$

$$= \frac{\overline{Y}_{st}}{2}E\left(2e_{0h} + e_{1h} + e_{0h}e_{1h} - 2\rho_h e_{1h} - 2\rho_h e_{0h}e_{1h} - \frac{e_{1h}^2}{4} + \rho_h e_{1h}^2\right) \quad (12.7)$$

$$= \frac{\overline{Y}_{st}}{2}\left(0 + 0 + \sum_{h=1}^{L}\theta_h W_h^2 \rho_h C_{xh}C_{yh} - 0 - 2\sum_{h=1}^{L}\theta_h W_h^2 \rho_h^2 C_{xh}C_{yh} - \frac{\sum_{h=1}^{L}\theta_h W_h^2 C_{xh}^2}{4} + \sum_{h=1}^{L}\theta_h W_h^2 \rho_h C_{xh}^2\right)$$

$$= \overline{Y}_{st}\sum_{h=1}^{L}\theta_h W_h^2\left(\frac{C_h C_{xh}^2}{2} - \rho_h^2 C_h C_{xh}^2 - \frac{C_x^2}{8} + \frac{\rho_h C_{xh}^2}{2}\right) \quad (12.8)$$

Therefore bias of the proposed estimator $\overline{y}^e_{RcP,st}$ to terms of order n^{-1} is obtained in as:

$$B\left(\overline{y}^e_{RcP,st}\right) = \overline{Y}_{st}\sum_{h=1}^{L}W_h^2\theta_h C_{xh}^2\left(\frac{C_h}{2} - \rho_h^2 C_h - \frac{1}{8} + \frac{\rho_h}{2}\right)$$

To obtain the MSE, let $MSE\left(\overline{y}^e_{RcP,st}\right) = E\left(\overline{y}^c_{RcP,st} - \overline{Y}_{st}\right)$ Squaring both sides of (12.8), and then taking expectation we obtain:
$MSE\left(\overline{y}^e_{RcP,st}\right) = E\left(\overline{y}^c_{RcP,st} - \overline{Y}_{st}\right)^2$

$$= E\left[\frac{\overline{Y}_{st}}{2}\left(2e_{0h} + e_{1h} + e_{0h}e_{1h} - 2\rho_h e_{1h} - 2\rho_h e_{0h}e_{1h} - \frac{e_{1h}^2}{4} + \rho_h e_{1h}^2\right)\right]^2 \quad (12.9)$$

$$= E\left[\frac{\overline{Y}^2_{st}}{4}\left(2e_{0h} + e_{1h} + e_{0h}e_{1h} - 2\rho_h e_{1h} - 2\rho_h e_{0h}e_{1h} - \frac{e_{1h}^2}{4} + \rho_h e_{1h}^2\right)\left(2e_{0h} + e_{1h} + e_{0h}e_{1h} - 2\rho_h e_{1h} - 2\rho_h e_{0h}e_{1h} - \frac{e_{1h}^2}{4} + \rho_h e_{1h}^2\right)\right]$$

$$= E\left[\frac{\overline{Y}^2_{st}}{4}\left(4e_{0h}^2 + 2e_{0h}e_{1h} - 4\rho_h e_{0h}e_{1h} + 2e_{0h}e_{1h} + e_{1h}^2 - 2\rho_h e_{1h}^2 - 4\rho_h e_{0h}e_{1h}^2 - 2\rho_h e_{1h}^2 + 4\rho_h^2 e_{1h}^2\right)\right]$$

$$= E\left[\frac{\overline{Y}_{st}^2}{4}\Big(4e_{0h}^2 + e_{1h}^2 + 4e_{0h}e_{1h} - 8\rho_h e_{0h}e_{1h} - 4\rho_h e_{1h}^2 + 4\rho_h^2 e_{1h}^2\Big)\right]$$

$$= \overline{Y}_h^2 \sum_{h=1}^{L} \theta_h W_h^2 \left(C_{yh}^2 + \frac{C_{xh}^2}{4} + \rho_h C_{xh}C_{yh} - 2\rho_h^2 C_{xh}C_{yh} - \rho_h C_{xh}^2 + \rho_h^2 C_{xh}^2\right)$$

$$= \overline{Y}_{st}^2 \sum_{h=1}^{L} \theta_h W_h^2 \left(C_{yh}^2 + C_{xh}^2\Big(\frac{1}{4} - \rho_h + \rho_h^2 + C_h - 2\rho_h C_h\Big) + C_h C_{xh}^2(1-2\rho_h)\right)$$

We obtain the MSE of the estimator $\overline{y}_{RcP,st}^e$ to terms of order n^{-1} as

$$MSE\Big(\overline{y}_{RcP,st}^e\Big) = \overline{Y}_{st}^2 \sum_{h=1}^{L} \theta_h W_h^2 \left(C_{yh}^2 + C_{xh}^2\Big(\frac{1}{4} - \rho_h + \rho_h^2 + C_h - 2\rho_h C_h\Big)\right) \qquad (12.10)$$

Table 3. **Appendix 2: Summary of the Study Population under Stratified Random Sampling**

	Stratum 1	**Stratum 2**	**Stratum 3**
$N = 244$	$N_1 = 183$	$N_2 = 50$	$N_3 = 11$
$n = 97$	$n_1 = 73$	$n_2 = 20$	$n_3 = 4$
$\bar{y} = 65268$	$\bar{y}_1 = 49384$	$\bar{y}_2 = 101250$	$\bar{y}_3 = 177500$
$\bar{x} = 78046$	$\bar{x}_1 = 58247$	$\bar{x}_2 = 122823$	$\bar{x}_3 = 242500$
$S_y = 43129$	$S_{y_1} = 22403.2675$	$S_{y_2} = 15206.9063$	$S_{y_3} = 68495.7420$
$S_x = 56906$	$S_{x_1} = 52016.2317$	$S_{x_2} = 25642.8634$	$S_{x_3} = 46457.8662$
$S_y^2 = 1860094072$	$S_{y_1}^2 = 501906393$	$S_{y_2}^2 = 231250000$	$S_{y_3}^2 = 4691666667$
$S_x^2 = 3238338524$	$S_{x_1}^2 = 2705688356$	$S_{x_2}^2 = 657556441$	$S_{x_3}^2 = 2158333333$
$S_{xy} = 2214844346$	$S_{x_1y_1} = 465015221$	$S_{x_2y_2} = 207944079$	$S_{x_3y_3} = 2308333333$
$\bar{Y} = 63045$	$\bar{Y}_1 = 50197$	$\bar{Y}_2 = 99700$	$\bar{Y}_3 = 158182$
$\bar{X} = 74889$	$\bar{X}_1 = 51656$	$\bar{X}_2 = 123220$	$\bar{X}_3 = 249091$
$C_y = 0.6841$	$C_{y_1} = 0.4463$	$C_{y_2} = 0.1525$	$C_{y_3} = 0.4330$
$C_x = 0.7599$	$C_{x_1} = 1.0070$	$C_{x_2} = 0.2081$	$C_{x_3} = 0.1865$
$C_y^2 = 0.4680$	$C_{y_1}^2 = 0.1992$	$C_{y_2}^2 = 0.0233$	$C_{y_3}^2 = 0.1865$
$C_x^2 = 0.5774$	$C_{x_1}^2 = 1.0140$	$C_{x_2}^2 = 0.0433$	$C_{x_3}^2 = 0.0348$
$C = 0.8120$	$C_1 = 0.1768$	$C_2 = 0.3906$	$C_3 = 1.6832$
$f = 0.3975$	$f_1 = 0.3989$	$f_2 = 0.4$	$f_3 = 0.3636$
$\theta = 0.006$	$\theta_1 = 0.00$	$\theta_2 = 0.035$	$\theta_3 = 0.1591$
$R = 0.8418$	$w_1 = 0.75$	$w_2 = 0.2049$	$w_3 = 0.0451$
$\rho = 0.902$	$R_1 = 0.9718$	$R_2 = 0.8091$	$R_3 = 0.6350$
	$\rho_1 = 0.399$	$\rho_2 = 0.533$	$\rho_3 = 0.725$

Source: *Survey of Household in Makurdi, Nigeria.*