# Robustness of GARCH family models to high positive autocorrelation

**Ngozi G. Emenogu[1] and Monday Osagie Adenomon[2,3*]**

[1]Department of Statistics, Federal Polytechnic, Bida, Niger State, Nigeria
[2]Department of Statistics, Nasarawa State University, Keffi, Nasarawa State, Nigeria
[3]Foundation of Laboratory for Econometrics and Applied Statistics of Nigeria (FOUND-LEAS-IN-NIGERIA)

*This study compared the performance of five Family Generalized Auto-Regressive Conditional Heteroscedastic (fGARCH) models (sGARCH, gjrGARCH, iGARCH, TGARCH and NGARCH) in the presence of high positive autocorrelation. To achieve this, financial time series was simulated with autocorrelated coefficients as ρ = (0.8, 0.85, 0.9, 0.95, 0.99), at different time series lengths (as 250, 500, 750, 1000, 1250, 1500) and each trial was repeated 1000 times carried out in R environment using rugarch package. The performance of the preferred model was judged using Root Mean Squared Error (RMSE) and Mean Absolute Error (MAE). Results from the simulation revealed that these GARCH models performances varies with the different autocorrelation values and at different time series lengths. But in the overall, NGARCH model dominates with 62.5% and 59.3% using RMSE and MAE respectively. We therefore recommended that investors, financial analysts and researchers interested in stock prices and asset return should adopt NGARCH model when there is high positive autocorrelation in the financial time series data.*

**Keywords:** financial Time Series; autocorrelation; Models; GARCH; RMSE; MAE.

## 1 Introduction

Financial time series analysis is mainly concerned with the theory and application of asset valuation over time, because of this financial time series analyses are very useful to financial analysts and portfolio managers (Tsay, 2005). Financial time series contains uncertainty, volatility, excess kurtosis, high standard deviation, high skewness and sometimes non normality (Pedroni, 2001 and Grigoletto and Lisi, 2009).

Volatility is a very important element of financial time series since its introduction by Engle (1982). Models such as Auto-Regressive Conditional Heteroscedastic (ARCH), Generalized Auto-Regressive Conditional Heteroscedastic (GARCH), multivariate GARCH, Stochastic volatility (SV) and various variants of the models have been proposed to handle volatility in financial time series (Lawrance, 2013). In fact, the ARCH and GARCH models are now so widely used and they are referred to as the "workhorse of the financial industry" (Lee and Hansen, 1994 and Lange, 2011).

Many economic time series including financial time series are strongly autocorrelated and can be modeled by linear (near) unit root or I(1) processes such as Threshold Autoregressive (TAR) model (Lanne and Saikkonen, 2002). Autocorrelation leads to serious underestimation of standard error for regression coefficients and makes prediction intervals to be excessively wide and as such the presence of autocorrelation renders inference and decision making about the estimated parameters invalid (Adenomon et al., 2015). This means

* Corresponding Author; E-mail: adenomonmo@nsuk.edu.ng

that financial time series and its possible modeling techniques such as ARCH and GARCH models may not also be immune against the effects of autocorrelation. The aim of this study is conducted to investigate the robustness of family GARCH models in the presence of high levels of autocorrelation.

## 2 Brief Review of Literature

### 2.1 Modelling financial Time Series with autocorrelation

Autocorrelation can be defined as correlation between members of series of observations ordered in time (as in time series data) or space (as in cross-section data) (Adenomon and Micheal, 2017). Gujarati (2003) identified the following as causes of autocorrelation in time series data: inertia, specification Bias, excluded variables, incorrect functional form, cobweb phenomenon, lags, data transformation and manipulation, and non-stationarity.

Lanne and Saikkonen (2002) investigated economic time series that are strongly autocorrelated using Threshold Autoregression because TAR model can accommodated time series that are strongly autocorrelated or I(1) time series processes. Xiao et al. (2003) proposed a modification of local polynomial time series regression estimators that improves efficiency when the innovation process is autocorrelated which was based on pre-whitening transformation of the dependent variable that must be estimated from the time series data. Their proposed method was more efficient than the conventional local polynomial method. Their study is similar to the work of Su and Ullah (2006). Lee and Lund (2004) investigated the properties of ordinary and generalized least squares in a simple linear regression with stationary autocorrelated errors. They derived variances of the parameter estimators for some time series autocorrelation structures which include a first order autocorrelation and general moving averages.

Lanne and Saikkonen (2005) investigated and proposed non-linear GARCH models for highly persistent volatility. They observed that conventional GARCH are inflexible to simultaneously first order autocorrelation of squares, persistence of shocks to volatility and excess kurtosis prevalent in financial return series. Lawrance (2013) examined volatility in financial time series using exploratory graphs and observed that volatility can be confused with the effect of negative autocorrelation and can be distorted by positive autocorrelation. Lawrence concluded that volatility can only be visualized and analyzed for linearly uncorrelated or decorrelated series.

## 3 Model Specification

This study focuses on the GARCH models that are robust for forecasting the volatility of financial time series data in the presence of high positive autocorrelation; so GARCH model and some of its extensions are presented in this section.

### 3.1 Autoregressive Conditional Heteroscedasticity (ARCH) Family Model

Every ARCH or GARCH family model requires two distinct specifications, namely: the mean and the variance equations (Atoi, 2014). The mean equation for a conditional heteroscedasticity in a return series, $y_t$, is given by

$$y_t = E_{t-1}(y_t) + \varepsilon_t, \quad (1)$$

where $\varepsilon_t = \phi_t \sigma_t$. The mean equation in equation (1) also applies to other GARCH family models, $E_{t-1}(.)$ is the expected value conditional on information available at time *t-1*, while $\varepsilon_t$ is the error generated from the mean equation at time t and $\phi_t$ is the sequence of independent and identically distributed random variables with zero mean and unit variance. The variance equation for an ARCH(p) model is given by

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2 \quad (2)$$

It can be seen in the equation that large values of the innovation of asset returns have bigger impact on the conditional variance because they are squared, which means that a large shock tends to follow another large shock and that is the same way the clusters of the volatility behave. So the ARCH(p) model becomes

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2, \quad (3)$$

where $\varepsilon_t \sim N(0,1) iid, \omega > 0$ and $\alpha_i \geq 0$ for $i > 0$. In practice, $\varepsilon_t$ is assumed to follow the standard normal or a standardized student-*t* distribution or a generalized error distribution (Tsay, 2005).

### 3.2 Asymmetric Power ARCH

According to Rossi (2004), the asymmetric power ARCH model proposed by Ding, Engle & Granger (1993) given below forms the basis for deriving the GARCH family models Given that

$$\begin{aligned} r &= \mu + a_t, \\ \varepsilon_t &= \sigma_t \varepsilon_t, \\ \sigma_t^\delta &= \omega + \textstyle\sum_{i=1}^{p} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \end{aligned} \quad (4)$$

where

$$\begin{aligned} &\omega > 0, && \delta \geq 0, \\ &\alpha_i \geq 0, && i = 1, 2, \cdots, p \\ &\beta_j > 0, && j = 1, 2, \cdots, q, \\ &-1 < \gamma_i < 1, && i = 1, 2, \cdots, p. \end{aligned}$$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative "shocks".

### 3.3 Standard GARCH(p, q) (sGARCH) Model

The mathematical model for the GARCH(p,q) model is obtained from equation (4) by letting $\delta = 2$ and $\gamma_i = 0, i = 1, \cdots, p$, we have

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \omega + \textstyle\sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \quad (5)$$

where $a_t = r_t - \mu_t$, ($r_t$ is the continuously compounded log return series), and $\varepsilon_t$~N(0, 1) *iid*, the parameter $\alpha_i$is the ARCH parameter and $\beta_j$ is the GARCH parameter, and $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{\max(p,q)}(\alpha_i + \beta_i) < 1$, (Rossi, 2004; Tsay, 2005 and Jiang, 2012). The restriction on ARCH and GARCH parameters $(\alpha_i, \beta_i)$, suggests that the volatility $(a_t)$, is finite and that the conditional standard deviation $(\sigma_t)$ increases. It can be observed that if q = 0, then the model GARCH parameter $(\beta_j)$ becomes extinct and what is left is an ARCH(p) model.

To expatiate on the properties of GARCH models, the following representation is necessary: Let $\eta_t = a_t^2 - \sigma_t^2$, so that $\sigma_t^2 = a_t^2 - \eta_t$. By substituting $\sigma_{t-i}^2 = a_{t-i}^2 - \eta_{t-i}$. $(i = 0, \cdots, q)$ into Eq. (4), the GARCH model can be rewritten as

$$a_t = \alpha_0 + \sum_{i=1}^{\max(p,q)}(\alpha_i + \beta_i)a_{t-i}^2 + \eta_t - \sum_{j=1}^{q}\beta_j\eta_{t-j} \quad . \tag{6}$$

It can be seen that $\{\eta_t\}$ is a martingale difference series (i.e.,$E(\eta_t) = 0$, and $cov(\eta_t, \eta_{t-j}) = 0$, for $j \geq 1$). However, $\{\eta_t\}$ in general is not an *iid* sequence. A GARCH model can be regarded as an application of the ARMA idea to the squared series $a_t^2$. Using the unconditional mean of an ARMA model, results in $E(a_t^2) = \frac{\alpha_0}{1-\sum_{i=1}^{\max(p,q)}(\alpha_i+\beta_i)}$, provided that the denominator of the prior fraction is positive (Tsay, 2005). When p =1 and q =1, we have GARCH(1, 1) model given by

$$a_t = \sigma_t\varepsilon_t,$$
$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \beta_1\sigma_{t-1}^2, \tag{7}$$

### 3.4 GJR-GARCH(p,q) Model

The Glosten-Jagannathan-Runkle GARCH (GJRGARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$. When $\delta = 2 \; and \; 0 \leq \gamma_i < 1$,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2,$$

$$= \omega + \sum_{i=1}^{p}\alpha_i(|\varepsilon_{t-i}|^2 + \gamma_i^2\varepsilon_{t-i}^2 - 2\gamma_i|\varepsilon_{t-i}|\varepsilon_{t-i}) + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 . \tag{8}$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p}\alpha_i^2(1+\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 , & \varepsilon_{t-i} < 0 \\ \omega + \sum_{i=1}^{p}\alpha_i^2(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2, & \varepsilon_{t-i} > 0 \end{cases}$$

That is, $\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i(1-\gamma_i)^2\,\varepsilon_{t-1}^2 + \sum_{i=1}^{p}\alpha_i\{(1+\gamma_i)^2\varepsilon_{t-i}^2 - (1 - \gamma_i)^2\}S_t^-\,\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p}\alpha_i(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-i}^2 + \sum_{i=1}^{p}4\alpha_i\gamma_i S_i^-\,\varepsilon_{t-i}^2$$

where $$S_i^- = \begin{cases} 1 & if\ \varepsilon_{t-i} < 0 \\ 0 & if\ \varepsilon_{t-i} \geq 0 \end{cases}$$

Now, define $\alpha_i^* = \alpha_i(1-\gamma_i)^2$, and $\gamma_i^* = 4\alpha_i\gamma_i$, then

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i^* S_i^- \varepsilon_{t-i}^2, \qquad (9)$$

which is the GJRGARCH model (Rossi, 2004). But when $-1 \leq \gamma_i < 0$, then, recall Eq. (8)

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}| - \gamma_i\varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2,$$

$$= \omega + \sum_{i=1}^{p} \alpha_i(|\varepsilon_{t-i}|^2 + \gamma_i^2\varepsilon_{t-i}^2 - 2\gamma_i|\varepsilon_{t-i}|\varepsilon_{t-i}) + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2$$

$$\sigma_t^2 = \begin{cases} \omega + \sum_{i=1}^{p} \alpha_i^2(1-\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j,}^2 & \varepsilon_{t-j} > 0 \\ \omega + \sum_{i=1}^{p} \alpha_i(1+\gamma_i)^2\,\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2, & \varepsilon_{t-i} < 0 \end{cases}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i(1+\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i\{(1-\gamma_i)^2 - (1+\gamma_i)^2\}S_i^+\varepsilon_{t-i}^2$$

$$= \omega + \sum_{i=1}^{p} \alpha_i(1+\gamma_i)^2\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-j}^2 + \sum_{i=1}^{p} \alpha_i\{1 + \gamma_i^2 - 2\gamma_i - 1 - \gamma_i^2 - 2\gamma_i\}S_i^+\varepsilon_{t-i}^2$$

where

$$S_i^+ = \begin{cases} 1, & if\,\varepsilon_{t-i} > 0 \\ 0, & if\,\varepsilon_{t-i} \leq 0 \end{cases}.$$

Also define $\alpha_i^* = \alpha_i(1+\gamma_i)^2$ and $\gamma_i^* = -4\alpha_i\gamma_i$. Then

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i^*\varepsilon_{t-i}^2 + \sum_{j=1}^{q} \beta_j\sigma_{t-i}^2 + \sum_{i=1}^{p} \gamma_i^* S_i^+ \varepsilon_{t-i}^2, \qquad (10)$$

which allows positive shocks to have a stronger effect on volatility than negative shocks (Rossi, 2004). But when $p = q = 1$, the GJRGARCH(1,1) model will be written as

$$\sigma_t^2 = \omega + \alpha t + \gamma S_i \varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \qquad (11)$$

### 3.5 IGARCH(1, 1) Model

The integrated GARCH (IGARCH) models are unit- root GARCH models. The IGARCH (1, 1) model is specified in Tsay (2005) and Grek (2014) as

$$a_t = \sigma_t\varepsilon_t; \quad \sigma_t^2 = \alpha_0 + \beta_1\sigma_{t-1}^2 + (1-\beta_1)a_{t-1}^2 \qquad (12)$$

Where $\varepsilon_t \sim N(0,1)\ iid$, and $0 < \beta_1 < 1$, Ali (2013) used $\alpha_i$ to denote $1 - \beta_i$. The model is also an exponential smoothing model for the $\{a_t^2\}$ series. To see this, rewrite the model as

$$\sigma_t^2 = (1-\beta_1)a_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

$$= (1-\beta_1)a_{t-1}^2 + \beta_1[(1-\beta_1)a_{t-2}^2 + \beta_1\sigma_{t-2}^2]$$

$$= (1-\beta_1)a_{t-1}^2 + (1-\beta_1)\beta_1 a_{t-2}^2 + \beta_1^2\sigma_{t-2}^2. \quad (13)$$

By repeated substitutions, we have

$$\sigma_t^2 = (1-\beta_1)(a_{t-1}^2 + \beta_1 a_{t-2}^2 + \beta_1^2 a_{t-3}^3 + \cdots) \quad (14)$$

which is the well-known exponential smoothing formation with $\beta_1$ being the discounting factor (Tsay, 2005).

### 3.6 TGARCH(p, q) Model

The Threshold GARCH model is another model used to handle leverage effects, and a TGARCH(p, q) model is given by the following:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t-i})^2 a_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2, \quad (15)$$

where $N_{t-i}$ is an indicator for negative $a_{t-i}$; that is,

$$N_{t-i} = \begin{cases} 1 & if\, \varepsilon_{t-i} < 0, \\ 0 & if\, \varepsilon_{t-i} \geq 0, \end{cases}$$

and $\alpha_i$, $\gamma_i$, and $\beta_j$, are nonnegative parameters satisfying conditions similar to those of GARCH models, (Tsay, 2005). When $p = 1, q = 1$, the TGARCH(1, 1) model becomes

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1})a_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (16)$$

### 3.7 NGARCH(p, q) Model

The Nonlinear Generalized Autoregressive Conditional Heteroscedasticity (NGARCH) Model has been presented variously in literature by the following scholars: Hsieh and Ritchken (2005), Lanne and Saikkonen (2005), Malecka (2014) and Kononovicius and Ruseckas (2015). The following model can be shown to represent all the presentations:

$$h_t = \omega + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{q}\gamma_i\varepsilon_{t-i} + \sum_{j=1}^{p}\beta_j h_{t-j}, \quad (17)$$

where $h_t$ is the conditional variance, and $\omega$, $\beta$ and $\alpha$ satisfy $\omega > 0, \beta \geq 0$ and $\alpha \geq 0$; which can also be written as

$$\sigma_t = \omega + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \sum_{i=1}^{q}\gamma_i\varepsilon_{t-i} + \sum_{j=1}^{p}\beta_j\sigma_{t-j}. \quad (18)$$

### 3.8 Skewed GARCH(p, q) Model

The Skewed GARCH model can be written as

$$Y_t = \eta_t\varepsilon_t$$

$$\eta_t^2 = \delta_0 + \sum_{i=1}^{q}\delta_i(\eta_{t-i}\varepsilon_{t-i})^2 + \sum_{j=q+1}^{q+p}\delta_j\eta_{t+q-j}^2 \quad (19)$$

where $Y_t$ is the leading market return at time $t$, $\{\varepsilon_t\}\sim N(0,1)\ iid$, is the innovation (or shock) of the market, and is hypothesized to be Gaussian. $\delta_0$ has to be positive and the remaining parameters nonnegative in order to ensure the positivity of $\eta_t^2$, (De Luca and Loperfido, 2012).

## 4 Materials and Methods

### 4.1 Simulation Procedure

The simulation procedure here considers the following equations of GARCH (1,1):

$$\varepsilon_t = \sigma_t z_t,$$

$$\sigma_t^2 = a_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (20)$$

The case simulated is the case of financial time series where there are positive high autocorrelation coefficients as ρ = (0.8, 0.85, 0.9, 0.95, 0.99), at different time series lengths (as 250, 500, 750, 1000, 1250, 1500). The experiment is repeated 1000 times. The rugarch package of the R software was used to execute the simulation.

### 4.2 Forecast Assessment

The following are the criteria for Forecast assessments used:

1. Mean Absolute Error (MAE) has a formula $MAE_j = \frac{\sum_{1=1}^{n}|e_i|}{n}$. This criterion measures deviation from the series in absolute terms, and measures how much the forecast is biased. This measure is one of the most common ones used for analyzing the quality of different forecasts.
2. The Root Mean Square Error (RMSE) is given as $RMSE_j = \sqrt{\frac{\sum_i^n (y_i - y^f)^2}{n}}$, where $y_i$ is the time series data and $y^f$ is the forecast value of y (Caraiani, 2010).

For the two measures above, the smaller the value, the better the fit of the model (Cooray, 2008). In this simulation study, $RMSE = \frac{\sum_j^N RMSE_j}{N}\ and\ MAE = \frac{\sum_j^N MAE_j}{N}$, where N=1,000, is the number of iterations or replications in the simulation study. Therefore, the model with the minimum RMSE and MAE result will be the preferred model.

## 5 Results

### 5.1 Simulation Analysis Results

The results of the simulation carried out are presented in Table 1 to Table 12 below. Ranking was done for each forecasting assessment criterion (that is RMSE and MAE). For instance, for RMSE, the least value of RMSE is ranked 1, next is ranked 2 and so on. The same process is also used for ranking using MAE value.

**Table 1: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.8 at different time series lengths**

| Autocorrelation Coefficient | 0.8 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 15.71621 | 198.40292 | 22.26565 | 397.21749 | 27.32206 | 596.84701 |
| gjrGARCH | 15.67723 | 197.67197 | 22.26607 | 397.35969 | 27.34346 | 597.34298 |
| iGARCH | 15.67723 | 197.67197 | 22.28668 | 397.33560 | 27.29018 | 596.33660 |
| TGARCH | *NA | *NA | 22.29444 | 397.54459 | 27.33909 | 597.47104 |
| NGARCH | *NA | *NA | 22.27423 | 397.56284 | 27.24909 | 595.71351 |

Table 1 Continued:

| Autocorrelation Coefficient | 0.8 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| gjrGARCH | 31.5664 | 796.4621 | 35.31191 | 996.36279 | 38.68945 | 1195.24246 |
| iGARCH | 31.54257 | 795.60796 | 35.2902 | 995.3027 | 38.6903 | 1195.2416 |
| TGARCH | 31.59317 | 796.72437 | 35.3148 | 995.9538 | 38.66294 | 1194.47858 |
| NGARCH | 31.55313 | 795.87381 | 35.32081 | 996.50479 | 38.7049 | 1195.9127 |
| | 31.53909 | 795.63060 | 35.23728 | 994.15621 | 38.59607 | 1192.49545 |

**Table 2: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.8 at different time series lengths**

| Autocorrelation Coefficient | 0.8 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 3.0 | 3.0 | 1 | 1 | 3 | 3 |
| gjrGARCH | 1.5 | 1.5 | 2 | 3 | 5 | 4 |
| iGARCH | 1.5 | 1.5 | 4 | 2 | 2 | 2 |
| TGARCH | *NA | *NA | 5 | 4 | 4 | 5 |
| NGARCH | *NA | *NA | 3 | 5 | 1 | 1 |

Table 2 Continued:

| Autocorrelation Coefficient | 0.8 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 4 | 4 | 3 | 4 | 3 | 4 |
| gjrGARCH | 2 | 1 | 2 | 2 | 4 | 3 |
| iGARCH | 5 | 5 | 4 | 3 | 2 | 2 |
| TGARCH | 3 | 3 | 5 | 5 | 5 | 5 |
| NGARCH | 1 | 2 | 1 | 1 | 1 | 1 |

Tables 1 and 2 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.8 at different time series lengths.

**Table 3: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.85 at different time series lengths**

| Autocorrelation Coefficient | 0.85 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 15.66386 | 197.70122 | 22.25821 | 396.90223 | 27.28654 | 595.80040 |
| gjrGARCH | 15.68658 | 197.91230 | 22.24494 | 396.80352 | *NA | *NA |
| iGARCH | 15.69286 | 197.90984 | 22.25257 | 396.79475 | 27.34184 | 597.43763 |
| TGARCH | 15.71391 | 198.18182 | 22.26876 | 397.13353 | 27.23526 | 595.14723 |
| NGARCH | 15.61822 | 197.17366 | 22.21827 | 396.37184 | 27.25388 | 595.46976 |

Table 3 Continued:

| Autocorrelation Coefficient | 0.85 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 31.55959 | 796.16952 | 35.31079 | 995.68791 | 38.72317 | 1196.32629 |
| gjrGARCH | 31.5704 | 796.1239 | 35.34999 | 997.37200 | *NA | *NA |
| iGARCH | 31.54111 | 795.49648 | 35.32417 | 996.32265 | 38.69188 | 1195.64099 |
| TGARCH | 31.57198 | 796.59430 | 35.33417 | 996.52832 | 38.72268 | 1196.43748 |
| NGARCH | 31.50466 | 795.42553 | 35.30166 | 995.55099 | 38.66997 | 1194.57390 |

**Table 4: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.85 at different time series lengths**

| Autocorrelation Coefficient | 0.85 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 2 | 2 | 4 | 4 | 3 | 3 |
| gjrGARCH | 3 | 4 | 2 | 3 | *NA | *NA |
| iGARCH | 4 | 3 | 3 | 2 | 4 | 4 |
| TGARCH | 5 | 5 | 5 | 5 | 1 | 1 |
| NGARCH | 1 | 1 | 1 | 1 | 2 | 2 |

Table 4 Continued:

| Autocorrelation Coefficient | 0.85 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 3 | 4 | 2 | 2 | 4 | 3 |
| gjrGARCH | 4 | 3 | 5 | 5 | *NA | *NA |
| iGARCH | 2 | 2 | 3 | 3 | 2 | 2 |
| TGARCH | 5 | 5 | 4 | 4 | 3 | 4 |
| NGARCH | 1 | 1 | 1 | 1 | 1 | 1 |

Tables 3 and 4 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.85 at different time series lengths.

**Table 5: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.9 at different time series lengths**

| Autocorrelation Coefficient | 0.9 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 15.69056 | 197.97735 | 22.22066 | 396.54629 | 27.34395 | 597.45662 |
| gjrGARCH | 15.69322 | 198.21389 | 22.2961 | 397.7557 | 27.36271 | 598.10197 |
| iGARCH | 15.67502 | 197.83526 | 22.29554 | 397.73077 | 27.23304 | 594.83387 |
| TGARCH | 15.6704 | 197.9100 | 22.27206 | 397.27891 | 27.34957 | 597.51156 |
| NGARCH | 15.66183 | 197.72987 | 22.25393 | 397.27571 | 27.25242 | 595.38001 |

Table 5 Continued:

| Autocorrelation Coefficient | 0.9 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 31.5213 | 795.3229 | 31.5213 | 795.3229 | 38.68276 | 1195.44711 |
| gjrGARCH | 31.5506 | 796.2314 | 31.5506 | 796.2314 | 38.69959 | 1196.23093 |
| iGARCH | 31.48481 | 794.08972 | 31.48481 | 794.08972 | 38.65638 | 1194.28637 |
| TGARCH | 31.55911 | 796.64856 | 31.55911 | 796.64856 | 38.64567 | 1194.20921 |
| NGARCH | *NA | *NA | *NA | *NA | 38.58901 | 1192.30893 |

**Table 6: The Ranks of The RMSE and MAE values from the fGARCH family model at for autocorrelation value of 0.9 at different time series lengths**

| Autocorrelation Coefficient | 0.9 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 4 | 4 | 1 | 1 | 3 | 3 |
| gjrGARCH | 5 | 5 | 5 | 5 | 5 | 5 |
| iGARCH | 3 | 2 | 4 | 4 | 1 | 1 |
| TGARCH | 2 | 3 | 3 | 3 | 4 | 4 |
| NGARCH | 1 | 1 | 2 | 2 | 2 | 2 |

Table 6 Continued:

| Autocorrelation Coefficient | 0.9 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 3.5 | 2 | 3.5 | 2 | 4 | 4 |
| gjrGARCH | 1 | 4 | 1 | 4 | 5 | 5 |
| iGARCH | 3.5 | 1 | 3.5 | 1 | 3 | 3 |
| TGARCH | 2 | 3 | 2 | 3 | 2 | 2 |
| NGARCH | *NA | *NA | *NA | *NA | 1 | 1 |

Tables 5 and 6 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.9 at different time series lengths.

**Table 7**: **The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.95 at different time series lengths**

| Autocorrelation Coefficient | 0.95 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 15.68814 | 197.73295 | 22.29211 | 397.51403 | 27.2964 | 596.6663 |
| gjrGARCH | 15.69865 | 198.10642 | 22.2407 | 396.9439 | 27.30838 | 597.00994 |
| iGARCH | 15.73973 | 198.55254 | 22.2631 | 397.3768 | 27.30615 | 596.77193 |
| TGARCH | 15.72409 | 198.23185 | 22.26515 | 397.06924 | 27.32061 | 596.60567 |
| NGARCH | 15.64798 | 197.42033 | 22.22985 | 396.67303 | 27.28743 | 596.51312 |

Table 7 Continued:

| Autocorrelation Coefficient | 0.95 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 31.61276 | 797.63204 | 35.35245 | 997.24819 | 38.64961 | 1194.17442 |
| gjrGARCH | 31.55876 | 796.28403 | 35.34023 | 996.77395 | 38.69062 | 1195.48262 |
| iGARCH | 31.52512 | 795.55465 | 35.32587 | 996.60224 | 38.71167 | 1196.78469 |
| TGARCH | 31.52324 | 795.34252 | 35.2704 | 995.3214 | *NA | *NA |
| NGARCH | 31.55345 | 795.67381 | 35.15163 | 991.66978 | 38.58982 | 1192.56306 |

**Table 8: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.95 at different time series lengths**

| Autocorrelation Coefficient | 0.95 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 2 | 2 | 5 | 5 | 2 | 3 |
| gjrGARCH | 3 | 3 | 2 | 2 | 4 | 5 |
| iGARCH | 5 | 5 | 3 | 4 | 3 | 4 |
| TGARCH | 4 | 4 | 4 | 3 | 5 | 2 |
| NGARCH | 1 | 1 | 1 | 1 | 1 | 1 |

Table 8 Continued:

| Autocorrelation Coefficient | 0.95 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 5 | 5 | 5 | 5 | 2 | 2 |
| gjrGARCH | 4 | 4 | 4 | 4 | 3 | 3 |
| iGARCH | 2 | 2 | 3 | 3 | 4 | 4 |
| TGARCH | 1 | 1 | 2 | 2 | *NA | *NA |
| NGARCH | 3 | 3 | 1 | 1 | 1 | 1 |

Tables 7 and 8 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.95 at different time series lengths.

**Table 9: The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.99 at different time series lengths**

| Autocorrelation Coefficient | 0.99 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 15.61949 | 196.78527 | 22.17187 | 395.70803 | 27.28797 | 596.21018 |
| gjrGARCH | 15.5150 | 195.4712 | 22.35713 | 398.76400 | *NA | *NA |
| iGARCH | 15.54559 | 195.92525 | 22.21787 | 396.31757 | 27.25111 | 595.66820 |
| TGARCH | 15.6229 | 196.6654 | 22.26547 | 397.25319 | 27.2632 | 595.5894 |
| NGARCH | 15.6246 | 197.1020 | 22.16014 | 395.53835 | 27.25505 | 595.26933 |

Table 9 Continued:

| Autocorrelation Coefficient | 0.99 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 31.49784 | 794.64301 | 35.34941 | 997.45191 | 38.65248 | 1194.47871 |
| gjrGARCH | 31.58383 | 796.63904 | 35.24889 | 994.76280 | 38.6506 | 1194.6269 |
| iGARCH | 31.61138 | 797.79248 | 35.24748 | 994.22704 | 38.6874 | 1195.7461 |
| TGARCH | 31.56081 | 795.91771 | 35.21394 | 993.53380 | 38.72278 | 1196.99541 |
| NGARCH | 31.48974 | 794.83949 | 35.10384 | 989.91744 | 38.64695 | 1193.96249 |

**Table 10: The Ranks of The RMSE and MAE values from the fGARCH family model for autocorrelation value of 0.99 at different time series lengths**

| Autocorrelation Coefficient | 0.99 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 250 | | 500 | | 750 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 3 | 4 | 2 | 2 | 4 | 4 |
| gjrGARCH | 1 | 1 | 5 | 5 | *NA | *NA |
| iGARCH | 2 | 2 | 4 | 4 | 1 | 3 |
| TGARCH | 4 | 3 | 3 | 3 | 3 | 2 |
| NGARCH | 5 | 5 | 1 | 1 | 2 | 1 |

Table 10 Continued:

| Autocorrelation Coefficient | 0.99 | | | | | |
|---|---|---|---|---|---|---|
| Time series length (T) | 1000 | | 1250 | | 1500 | |
| Model | RMSE | MAE | RMSE | MAE | RMSE | MAE |
| sGARCH | 2 | 1 | 5 | 5 | 3 | 3 |
| gjrGARCH | 4 | 4 | 4 | 4 | 2 | 2 |
| iGARCH | 5 | 5 | 3 | 3 | 4 | 4 |
| TGARCH | 3 | 3 | 2 | 2 | 5 | 5 |
| NGARCH | 1 | 2 | 1 | 1 | 1 | 1 |

Tables 9 and 10 presents the RMSE and MAE values, and their respective ranks from the fGARCH family model for autocorrelation value of 0.99 at different time series lengths.

**Table 11: Summary of the Performances of the fGARCH family models at different levels of autocorrelation and time series lengths using RMSE**

| Forecast Statistics: RMSE | | | | | | |
|---|---|---|---|---|---|---|
| Autocorrelation Coefficient | Time series length (T) | | | | | |
| | 250 | 500 | 750 | 1000 | 1250 | 1500 |
| 0.8 | gjrGARCH iGARCH | sGARCH | NGARCH | NGARCH | NGARCH | NGARCH |
| 0.85 | NGARCH | NGARCH | TGARCH | NGARCH | NGARCH | NGARCH |
| 0.9 | NGARCH | sGARCH | iGARCH | gjrGARCH | gjrGARCH | NGARCH |
| 0.95 | NGARCH | NGARCH | NGARCH | TGARCH | NGARCH | NGARCH |
| 0.99 | gjrGARCH | NGARCH | iGARCH | NGARCH | NGARCH | NGARCH |

**Table 12: Summary of the Performances of the fGARCH family models at different levels of autocorrelation and time series lengths using MAE**

| Forecast Statistic: MAE | | | | | | |
|---|---|---|---|---|---|---|
| Autocorrelation Coefficient | Time series length (T) | | | | | |
| | 250 | 500 | 750 | 1000 | 1250 | 1500 |
| 0.8 | gjrGARCH iGARCH | sGARCH | NGARCH | gjrGARCH | NGARCH | NGARCH |
| 0.85 | NGARCH | NGARCH | TGARCH | NGARCH | NGARCH | NGARCH |
| 0.9 | NGARCH | sGARCH | iGARCH | iGARCH | iGARCH | NGARCH |
| 0.95 | NGARCH | NGARCH | NGARCH | TGARCH | NGARCH | NGARCH |
| 0.99 | gjrGARCH | NGARCH | NGARCH | sGARCH | NGARCH | NGARCH |

**Table 13: Overall performance rating of the fGARCH models irrespective of the autocorrelation values and time series lengths**

| Models | RMSE | MAE |
|---|---|---|
| sGARCH | 6.25% | 9.38% |
| gjrGARCH | 12.5% | 9.38% |
| iGARCH | 12.5% | 15.6% |
| TGARCH | 6.25% | 6.25% |
| NGARCH | 62.5% | 59.3% |

## 5.2 Discussion of Findings

### 5.2.1 GARCH models performance in the presence of autocorrelation using the Root Mean Square Error (RMSE) from the results of the simulation in Table 11.

For autocorrelation value of 0.8, iGARCH and gjrGARCH performed better than other models when the time series length (T) is 250, but at time series length (T) is 500, sGARCH performed better. However, NGARCH dominated as it outperformed the other models at the time series lengths (T) of 750, 1000, 1250 and 1500. For autocorrelation value of 0.85, NGARCH outperformed the other models irrespective of the time series (T) length except at time series length of 750 where TGARCH performed better than others. Coming to the autocorrelation value of 0.9, it can be seen that while NGARCH performed better at T = 250 and T = 1500, gjrGARCH performed better at T = 1000 and T = 1250, and sGARCH and iGARCH, respectively, outperformed the other models at T = 500 and T = 750. For autocorrelation value of 0.95, it can clearly be observed that NGARCH outperformed the other models irrespective of the time series (T) length except at time series length of 1000 where TGARCH performed better than others. For autocorrelation value of 0.99, while gjrGARCH performed better than other models at T = 250, iGARCH performed better than other models at T = 750, NGARCH outperformed the other models at the time series (T) lengths of 500, 1000, 1250 and 1500; thereby being the dominant model for that autocorrelation value.

### 5.2.2 GARCH models performance in the presence of autocorrelation using the root Mean Absolute Error (MAE) from the results of the simulation in Table 12

For autocorrelation value of 0.8, gjrGARCH and iGARCH performed better than other models when the time series length (T) is 250, gjrGARCH again performed better than other models at T = 1000, sGARCH outperformed the other models at T = 500, while NGARCH performed better than the other models at the time series lengths (T) of 750, 1250 and 1500. For autocorrelation value of 0.85, NGARCH dominated as it outperformed the other models irrespective of the time series (T) length except at time series length of 750 where TGARCH performed better than others. And for the autocorrelation value of 0.9, it can be seen that whereas NGARCH performed better at time series length (T) = 250 and T = 1500, sGARCH performed better than the other models at T = 500, while iGARCH dominated at the other time series lengths, performing better at T = 750, T = 1000 and T = 1250.

For autocorrelation value of 0.95, it can be observed that NGARCH was the preferred model as it outperformed the other models irrespective of the time series (T) length except at time series length (T) of 1000 where TGARCH performed better than others. And for autocorrelation value of 0.99, while NGARCH dominated the other models, performing better at time series lengths (T) = 500, T = 750, T = 1250 and T = 1500, gjrGARCH performed better than other models at T = 250, and sGARCH performed better than other models at T = 1000. Using RMSE and MAE criteria, this study has shown that different models performed better at different autocorrelation coefficients and at different time series lengths. This is in line with previous studies: Atoi (2014) in modelling the volatility of stock returns using daily closing data of Nigerian Stock Exchange, found that GARCH (1,1), PGARCH (1,1,1) and EGARCH (1,1) with student's t distribution, and TGARCH with GED were the four best fitted models based on Schwarz Information Criterion; and the conclusions in Grek (2014), Chen, Min and Chen (2013), Dijk, Franses and Lucas (1999) and Demos (2000), that different models performed differently under different conditions, and in this case, different autocorrelation coefficients and different time series lengths. Results are however in contrast to Mikosch and Starica (2000) using GARCH (1,1) model to estimate log return of foreign exchange, said that the level of autocorrelation is unreliable means for model selection. In summary in Table 13, modeling financial time series with high positive autocorrelation values is dominated by NGARCH model which is in line with results obtained by Lanne and Saikkonen (2005).

## 6 Conclusion

This study investigated the performance of fGARCH family models in the presence of high positive autocorrelation values. The findings revealed that sGARCH, gjrGARCH, iGARCH, TGARCH and NGARCHmodels performed at the rate of 6.25%, 12.5%, 12.5%, 6.25% and 62.5% respectively using RMSE. In addition, the findings also revealed that sGARCH, gjrGARCH, iGARCH, TGARCH and NGARCH models performed at the rate of 9.38%, 9.38%, 6.25% and 59.3% respectively using MAE. In the overall, TGARCH model performed poorly in the presence of high autocorrelation while NGARCH model dominated in the presence of high levels of autocorrelation. We therefore recommended that investors, financial analysts and researchers interested in stock prices and asset return should adapt NGARCH model when there is high positive autocorrelation in the financial time series data.

## Reference


Adenomon, M.O. and Micheal, V.A. (2017). *Introduction Econometrics with Examples in EViews*. Jube-Evans Book & Publication, Niger State.

Adenomon, M.O., Micheal, V.A. and Evans, P.O. (2015). A Simulation Study on the Performances of Classical VAR and Sims-Zha Bayesian VAR Models in the Presence of Autocorrelated Errors, *Open Journal of Modeling and Simulation*, 3, 146-158.

Ali, G. (2013). EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH, and APARCH Models for Pathogens at Marine Recreational Sites, *Journal of Statistical and Econometric Methods*, 2 (3), 57-73.

Atoi, N.V. (2014). Testing Volatility in Nigeria Stock Market Using GARCH Models, C*BN Journal of Applied Statistics*, 5, 65-93.

Caraiani, P. (2010). Forecasting Romanian GDP using a BVAR model, *Romanian Journal of Economic Forecasting*, 4, 76-87.

Chen, S., Min, W. and Chen, R. (2013). Model Identification for Time Series with Dependent Innovations, *Statistica Sinica*, 23, 873 - 899.

Cooray, T.M.J.A. (2008), *Applied Time series Analysis and Forecasting,* Narosa Publishing House, New Delhi.

De Luca, G. and Loperfido, N. (2012). Modeling Multivariate Skewness in Financial Returns: A SGARCH Approach, *The European Journal of Finance*, 1 -19, iFirst. http://dx.doi.org/10.1080/1351847X.2011.640342.

Demos, A. (2000). Autocorrelation Function of Conditionally Heteroskedastic in Mean Models, Athens University of Economics and Business*, Working Paper,* 1- 52. Available at http://www2.aueb.gr/users/demos/WorkingPapers/hetln-dp.pdf .

Dijk, D.V., Franses, P.H. and Lucas, A.A. (1999). Testing for ARCH in the Presence of Additive Outliers, *Journal of Applied Econometrics*, 14 (5), 539-562.

Ding, Z. Granger, C.W.J. and Engle, R.F. (1993). A Long Memory Property of Stock Market Returns and a New Model, *Journal of Empirical Finance*, 1, 83-106.

Engle, R.F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation, *Econometrica*, 50, 987-1007.

Grek, A. (2014). Forecasting Accuracy for ARCH Models and GARCH(1,1) Family which Model does Best Capture the Volatility of the Swedish Stock Market? *Statistics Advance Level Theses 15hp*, Örebro University.

Grigoletto, M. and Lisi, F. (2009). Looking for Skewness in Financial Time Series, *The Econometrics Journal*, 12(2), 310-323.

Gujarati, D.N. (2003). *Basic Econometrics*, 4th edition, The McGraw-Hill co, New Delhi.

Hsieh, K.C. and Ritchken, P. (2005). An Empirical Comparison of GARCH Option Pricing Models, *Review of Derivatives Research*, 8 (3), 129 -150.

Jiang, W. (2012). Using the GARCH Model to Analyse and Predict the Different Stock Markets, Masters' Degree Dissertation, Department of Statistics, Uppsala University, Sweden.

Kononovicius, A. and Ruseckas, J. (2015). Nonlinear GARCH Model and 1/f Noise. ARXIV:1412.6244V2[Q-FIN.ST].

Lange, T. (2011). Tail Behavior and OLS Estimating in AR-GARCH Models, Statistica Sinica, 21(3),1191-1200.

Lanne, M. and Saikkonen, P. (2002). Threshold Autoregression for Strongly Autocorrelated Time Series, 20(2), 282-289.

Lanne, M. and Saikkonen, P. (2005). Nonlinear GARCH Models for Highly Persistent Volatility. *Econometrics Journal*, 8 (2), 251-276.

Lawrance, A.J. (2013). Exploration Graphics for Financial Time Series Volatility, *Journal of the Royal Statistical Society, Series C*, 62(5), 669-686.

Lee, J. and Lund, R. (2004). Revisiting Simple Linear Regression with Autocorrelated Errors, *Biometrika*, 91(1), 240-245.

Lee, S. and Hansen, B. (1994). Asymptotic Theory for the GARCH(1,1) Quasi-Maximum Likelihood Estimator, *Econometric Theory*, 10, 29-53.

Malecka, M. (2014). GARCH Class Models Performance in Context of High Market Volatility, *ACTA Universitatis Lodziensis Folia Oeconmica*, 3, 253 -266.

Mikosch, T. and Starica, C. (2000). Limit Theory for the Sample Autocorrelations and Extremes of a GARCH(1,1) Process, *Annals of Statistics*, 28 (5), 1427 - 1451.

Pedroni, P. (2001). The Econometric Modeling of Financial Time Series by Terence Mills, *Journal of the American Statistical Association*, 96(453), 345-346.

Rossi, E. (2004). *Lecture notes on GARCH models*, University of Pavia, March.

Su, L. and Ullah, A. (2006). More Efficient Estimation in Nonparametric Regression with Non Parametric Autocorrelated Errors, *Econometric Theory*, 22(1), 98-126.

Tsay, R.S. (2005). *Analysis of financial time series*, 2nd Edition, John Wiley& Sons, New Jersey.

Xiao, Z. Linton, O.B., Carroll, R.J. and Mammen, E. (2003). More Efficient Local Polynomial Estimation in Nonparametric Regression with Autocorrelated Errors, *Journal of the American Statistical Association*, 98(464), 980-992.