Assessment of composite mixed resolution designs in spherical regions

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Composite Mixed Resolution Designs (CMRD) were evaluated using some design assessment criteria: D- and G-efficiencies, I-optimality and Condition Number in spherical regions. The effect of centre points, as well as the effect of the replication of the cube and star portions of the designs, were also evaluated. The CMRDs achieved high G- and D-efficiencies with zero or one centre points and minimum I-optimality criterion with four centre points. Also, replication of the cube and star portions of the designs improved the I-optimality criterion but did not improve the D- and G-efficiencies and the Condition Number.

Keywords: axial distance; centre point; minimum aberration; optimality criteria; replication

1. Introduction

Composite Mixed Resolution Design (CMRD) is a response surface design developed by Borkowski and Lucas (1997) which allows the estimation of interaction and quadratic effects among controllable (signal) variables in the presence of noise variables. The CMRD accommodates two sets of factors, namely; the signal factors (x) and noise factors (z). Signal factors are factors whose levels are easy to control in the process, whereas the levels of noise factors, which are hard or difficult to control, are assumed to vary at random within the process. The CMRD, like the CCD, has three distinct components, which include the factorial portion, the axial portion and the centre points.

The factorial portion is a 2^{K-p} full (p = 0) or fractional (p> 0) factorial mixed resolution design with c signal and u noise factors and with levels coded as ±1 (where K is the number of factors and p is the fraction of the factors). The factorial portion is a mixed resolution design because it has at least Resolution V among the c signal factors, at least Resolution III among the u noise factors and none of the $c \times u$ signal-by-noise two-factor interactions are aliased with any main effect or any two-factor interaction. The axial portion consists of $2c$ star points. For each signal factor the design includes two "star" points; that is, the signal factor is set at levels $\pm\alpha$ and all other factors are set at mid-level 0. This means that the star points for the noise factors are excluded from the design. n_0 centre points make up the third component of the design. The two parameters in the design that must be specified are the distance, α, of the axial runs from the design centre and the number of centre points, n_0.

When the experimental goal is specifically to acquire information that can lead to achieving a robust process, each design variable is classified as a member of the Signal variables or Noise variables. The signal or controllable variables are easy to control in the process while the noise variables are variables that are, in general, external to the process and are difficult or impossible to control (Borkowski and Lucas, 1997). It is well-known that variation in key performance characteristics can result in poor product and process quality

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(Myers et al., 2009). Most of the variabilities associated with such response can be attributed to the presence of the noise/uncontrollable factors. Robustness of a process involves reduction in process variability attributable to changes in the noise factors.

The introduction of Taguchi crossed-array design showed that experimental design and other statistical tools could be applied to the robustness problems in many cases. The Taguchi crossed-array design is used for reducing variations in products and processes or designing systems that are insensitive/robust to noise/uncontrollable factors that transmit major variability in a given product. The Taguchi system of experimental design for studying the robustness of a process revolved around the use of a statistical design for the controllable variables and another statistical design for the noise variables. However, studies have shown that Taguchi's methods are often inefficient: see, for example, Myers et al. (1992) and Montgomery (2013). One of the concerns is related to the absence of the means to test for higher-order signal factor interactions when Taguchi's crossed-arrays are used as inner-arrays for the design. In general, estimation of the complete set of interactions among the experimental variables is not possible when a response surface analysis is applied to data collected from a Taguchi crossed-array (Shoemaker et al., 1991).

Response surface method emerged as an alternative to the Taguchi crossed-array to address the many criticisms of the Taguchi method, by focusing on model building. The concept of model building lends itself to the use of response surface methodology (RSM) as an appealing approach to the robust design problem (Robinson and Wulff, 2006). Myers et al. (1992) suggested the use of single-factor (combined) arrays to be more economical than the crossed-array design. Combined array design is a type of design that has both the control factors and the noise factors in the same design matrix. The single-factor arrays can be used to provide information on process variability and yet allow a more reasonable approach to modeling the signal variables. Combined-arrays offer considerably more flexibility in the estimation of effects (Myers et al., 2009). Shoemaker et al. (1989) pointed out many illustrations in which the combining of the control and noise variables into a common array is far superior to crossing the inner and outer array as Taguchi advocates and also demonstrates the flexibility and usefulness of the combined-array for choosing a crossed-array with mixed resolution which allows for estimation of the effects of interest in RPD.

Myers et al. (1992) reviewed the principles that motivated the methods advocated by Taguchi and Wu (1980) for solution of a certain type of important industrial problems and they provided a concise account of an attractive alternative approach that is based on similar principles. They also itemized the five important general criticisms of Taguchi's contributions. Robinson and Wulff (2006) reviewed the applications of response surface methods to robust parameter designs and presented various experimental designs for fitting first- and second-order models with emphasis on the number of required runs and the terms that can be modeled. Some of the designs they reviewed include 3^k factorial design, Central Composite design, Modified Central Composite design(MCCD), Modified Small Composite design (MSCD) and Composite Mixed Resolution Designs (CMRD). Borkowski and Lucas (1997) developed the CMRD by redefining the model that was used for the Taguchi designs. That is, the designs are at least of resolution V among the signal factors such that, among the *C* signal factors, no main effects or two-factor interactions are aliased with any other main-effect or two-factor interaction. The designs are at least of resolution III among the noise factors. The experimental design region of interest in Borkowski and Lucas (1997) is a hypercube, where α, the axial distance, is 1.

In this study, we evaluate the performance characteristics of the Composite Mixed Resolution Designs in the spherical regions. Each design (see Table 1) was evaluated under the spherical axial distance, $S_\alpha = \sqrt{K}$, and the practical axial distance, $P_\alpha = \sqrt[4]{K}$, and the characteristics assessed using the D- and G-efficiencies and I-optimality criteria and condition number. There are 4 to 8 experimental factors considered with varying resolutions for $n_0 = 0$ to 4 centre points. In addition, the cube and star portions of the CMRDs were replicated to examine how such practice could improve the quality of the design. The replication procedure was conducted in the spherical regions.

2 Model development

The model for fitting the second-order response surface Composite Mixed Resolution Design is given by

$$y_{ijk} = \beta_0 + \sum_{i=1}^{c}\beta_i x_i + \sum_{i=1}^{c}\beta_{ii}x_i^2 + \sum_{i=1}^{c-1}\sum_{j=i+1}^{c}\beta_{ij}x_i x_j + \sum_{k=1}^{u}\delta_k z_k + \sum_{i=1}^{c}\sum_{k=1}^{u}\delta_{ik}x_i z_k + e_{ijk} \tag{1}$$

where y_{ijk} is the response, the β's and δ's are parameters, $\{x_i : i = 1, 2, \ldots, c\}$ and $\{z_k : k = 1, 2, \ldots, u\}$ are the signal and noise variables, respectively, and e_{ijk} is the random error term. The CMRD model excludes the interaction and quadratic effects of the noise variables thereby reducing the number of runs (this is because the axial portion of the noise variables is ignored) thereby, minimizing variability due to noise variables.

At a point (x, z) in the design space, the prediction variance is given by $Var[\hat{y}(\mathbf{x},\mathbf{z})] = \sigma^2 x'(X'X)^{-1}x$, where $x' = [1; x_1, x_2, \ldots, x_c; x_1^2, x_2^2, \ldots, x_c^2; x_1x_2, \ldots, x_{c-1}x_c; z_1, \ldots, z_u; x_1z_1, \ldots, x_cz_u]$ is the vector of design points in the design space expanded to model form and X is the design matrix. The prediction variance is scaled by multiplying by N, the total number of runs, to obtain the scaled prediction variance (SPV) of equation (2). Dividing by σ^2, the process variance, removes the unknown term.

$$\frac{NVar[\hat{y}(\mathbf{x},\mathbf{z})]}{\sigma^2} = Nx'(X'X)^{-1}x \tag{2}$$

The benefits of the scaled prediction variances will be explored in evaluating the performances of the CMRD.

The G-efficiency is one of the single-value design evaluation criteria that are based on the scaled prediction variance property of the design. The G-efficiency uses the maximum SPV, $\max\{Nx'(X'X)^{-1}x\}$, in design evaluation and is therefore, given by $G_{eff} = 100p/\max\{Nx'(X'X)^{-1}x\}$. The G-efficiency intuitively protects the experimenter against the worst-case scenario of the prediction variance being too undesirable since the user may wish to predict new responses anywhere in the design space and will be used in evaluating the CMRDs. The second single-value criterion that uses the scaled prediction variance in design evaluation is the *I*-optimality criterion which minimizes the normalized integrated scaled prediction variance, $I_{opt} = \min \frac{N}{\Psi} \int_R Var[\hat{y}(\mathbf{x},\mathbf{z})] d\mu(x,z)$, where $\mathbf{\Psi} = \int_{\Omega} dx$ is the volume of *R*, the design space and μ is uniform measure on R with total measure 1. The integral was simplified by Box and Draper (1963) such that $I_{opt} = N\{trace(MM_{\Omega}^{-1})\}$ where M is the moment matrix of the region of interest (Hardin and Sloane, 1993) and $M_{\Omega}^{-1} = (X'X)^{-1}$ is the inverse of the information matrix of the design, Ω.

Other criteria utilized in this study for the assessment of the composite mixed-resolution designs are the *D*-efficiency and the Condition number. The *D*-efficiency makes use of the determinant, $|X'X|$, of the information matrix. Under the standard normality assumptions, $|X'X|$ is inversely proportional to the square of the volume of the confidence region of the regression coefficients. The volume of the confidence region is relevant because it reflects how well the set of coefficients are estimated. Therefore, the larger the value of $|X'X|$, the better the estimation of the model parameters. The *D*-efficiency, given by $D_{eff} = \{100 \times |X'X^{1/p}|\}/N$ is a useful tool for quantifying the quality of the estimated model parameters. The power, $1/p$, takes account of the *p* parameter estimates being assessed when the determinant of the information matrix is being computed.

Condition Number is a measure of sphericity of the design (Leiviskä, 2013). If we describe the design as a matrix, *X*, consisting of -1's and +1's, the condition number is the ratio of the largest and smallest eigenvalues of the information matrix, *X'X*. All factorial designs without centre points (midpoints) have a condition number 1. The condition number, given by $\kappa(X) = [e_{\max}(X'X)/e_{\min}(X'X)]^{1/2}$, where $e_{\max}(X'X)$ and $e_{\min}(X'X)$ are the largest and the smallest eigenvalues is a measure of how close a matrix is to being singular. A matrix with large condition number is nearly singular, whereas a design whose information matrix has condition number that is close to 1 is far from being singular and the better the design. A non-singular matrix and its inverse have the same condition number.

The table of the generators of the designs to be used in this work is given in Table 1. In Table 1, the designs are labeled according to the total number of design factors, K = 4, …, 8 and reference letters, A, B, . . ., H. The generators/defining relations for these designs are found in Table 3. A generator (e.g. E = ABCD) is a set of columns (representing some factors) that are used to generate other factors that have not been in the fraction. For instance, for design 2, the fractional factorial required for the designs is 2^{5-1}. Therefore, $5 - 1 = 4$ factors (ABCD) is generated normally but the remaining factor (E) is generated with the generator (E = ABCD) by multiplying the ABCD columns together to result to the E column. The letters in the "Signal factors" and "Noise factors" columns (excluding the letters in bracket), correspond to the $K - p$ columns generating the full factorial design in $K - p$ factors. The levels of the remaining p factors (letters in the bracket) are generated using the *p* generators in the last column of Table 3. For example, Design 4 has $c = 3$ signal factors and $u = 4$ noise factors. The factorial portion of the CMR design is based on the 2^{7-2} fractional factorial design. The fractional factorial $K - p = 5$ factors (ABCDE) is generated normally while the levels of the remaining $p = 2$ factors (FG) is generated using the two generators (F = ABCE, G = ABCD).

Table 1: Generators of MAMR Designs

Design	K	Factors: Signal factors (SF)	Factors: Noise factors (NF)	Fraction	Resolution(s) SF	Resolution(s) NF	Generator(s)/Defining relation(s)
1	4	AB	CD	2^4	---	---	---
2	5	ABC	D(E)	2^{5-1}	V	V	E = ABCD
3	6	ABCD	E(F)	2^{6-1}	IV	IV	F = ABCDE
4	7	ABC	DE(FG)	2^{7-2}	V	IV	F = ABCE, G = ABCD, EFG
5	7	ABCD	E(FG)	2^{7-2}	V	III	F = ABCD, G =ABCDE, EFG
6	7	ABCDE	F(G)	2^{7-1}	VII	VII	G = ABCDEF
7	8	AB	CDE(FGH)	2^{8-3}	V	IV	F = ABCE, G = ABCD, H = ABDE, DEFG, CDFH, CEGH, ABFGH
8	8	ABC	DE(FGH)	2^{8-3}	V	III	F = ABCE, G = ABCD, H = ABCDE, DEFG, EGH, DFH, ABCFGH
9	8	ABCDEF	(GH)	2^{8-2}	V	V	G = CDEF, H = ABEF, ABCDGH

3 Design Comparisons

In this section, the CMR designs are compared with one another using the D and G-efficiencies, I-optimality criterion and condition number. The effect of the centre points and replication of cube and star portion could be seen from the results in Tables 2 and 3. Replication of experimental observations is considered indispensable for efficient and optimal performance of the second-order designs. Traditionally, the centre point of the design is replicated to ensure proper estimation of the experimental error with n_0 - 1 degrees of freedom as it is assumed that the optimum response is at the centre of the design. However, recent researches have shown that replicating at the centre alone may lead to estimating error that may be too small for correct evaluation of the model. Since there is no assurance that variability will remain constant throughout the design region, Dykstra (1960) posits that it is sound experimental strategy to replicate at other locations in the design region. See also, Giovannitti-Jensen and Myers (1989) for further recommendations on replication at other design locations apart from the centre point. Several works on replicating at other design locations have been focused on the central composite design (CCD). Such works include Dykstra (1960), Draper (1982), Borkowski (1995), Borkowski and Valeroso (2001), Chigbu and Ohaegbulem (2011) and Ukaegbu and Chigbu (2015). In this study, we also extend the replication of cube and star portions to the Composite Mixed Resolution designs, under spherical design region. For each replication of the cube portion, the star portion is not replicated and for each replication of the star portion, the cube portion is not replicated. Five versions of the designs are generated by replicating the cube and star portions by different amount. The first design is where the cube is not replicated and the star is not replicated. This design is denoted by C_1S_1. The second is C_1S_2, where the star is replicated twice and the cube is not replicated. Other designs areC_1S_3, C_2S_1 and C_3S_1. These designs are generated for each of the second-order CMRD designs for 4 to 8 factors. The nine (9) designs to be used in this study were taken from the table of Minimum Aberration Mixed Resolution (MAMR) designs in Borkowski and Lucas (1997) and presented in Table 1.

3.1 Comparison using G-efficiencies

We take a general view of the results in Tables 2 and 3 for the *G*-efficiency. It could be observed that increase in centre points of most of the designs does not improve the G-efficiencies of the designs. This is true for all designs with practical axial distances except design 6, where the best design has four centre points. But for designs with spherical axial distances, the best results for G-efficiency are achieved with 0 centre point except designs 1, 3 and 4 with their best results for G-efficiency achieved with one centre point and designs 6 and 9 achieved with two centre points. Also, replication of the cube and star portions of the designs does not improve the G-efficiency of the designs both for the spherical and practical axial distances. This is true for all except for design 6 where C_1S_2 has the highest *G*-efficiency. In general, the highest values of the *G*-efficiency are achieved when there are no replications of the cube and star. Note also that the G-efficiency values when there are replications of the star portion are higher than when the cube portion is replicated.

3.2 Comparison using D-efficiencies

Tables 2 and 3 show that all the designs with spherical axial distances, except designs 6 and 9, achieved the highest D-efficiencies values with zero centre points. Increasing the centre points tends to reduce the values of the D-efficiencies. Though, most of the designs with practical axial distance, which include designs 2, 3, 7 and 8 achieved the highest *D*-efficiency values with one centre point while designs 1 and 4 have the highest *D*-efficiency values achieved with two centre points. Also, designs 2 and 4 with spherical axial distance have *D*-efficiency values greater than 100% and should be regarded as super *D*-efficient designs. The *D*-efficiency values of the designs with spherical axial distances are more efficient than those with practical axial distance. Replicating the cube and star portions of most of the designs tends to reduce the values of the *D*-efficiency. This means that replication of cube and star portion consistently does not improve the *D*-efficiencies of the designs both for the spherical and practical axial distances. The exceptions are designs 2, 3 and 4.

3.3 Comparison using I-optimality Criterion

In contrast to the results of the *G*- and *D*-efficiencies, the *I*-optimality values of all the composite mixed resolution designs reduce consistently as the centre point increases. That is, increasing the centre points improves the values of the *I*-optimality criterion for all designs. Therefore, the smallest *I*–optimality values are achieved with four centre points for both the spherical and practical axial distances. Also, replicating the cube and star portions of the designs consistently reduces the *I*-optimality values for both the spherical and practical axial distances. The only exceptions are designs 3 and 6 with practical axial distance where the smallest values of the I-optimality criterion is achieved with the star replicated three times.

3.4 Comparison using Condition number

For the Condition Number of the designs, the results consistently show that as the number of centre points increases, the Condition Number improved. This holds consistently for all the designs, both for spherical and practical axial distances. For the spherical axial distance, replication of the cube and star portions of most of the designs, except design 6, does not improve the Condition Number. This is because the condition numbers for those designs increases with increase in the cube and star replication of the designs. But for the designs with practical axial distances, designs 3, 5, 6 and 7 have their best results achieved when the star portion is replicated while for designs 1, 2, 4 and 8, replication of the cube and star portion does not improve the Condition Number.

4. Conclusion

From the foregoing, most of the designs achieved their best D and G-efficiencies values without centre points and in some cases with one centre point. However, for the *I*-optimality criterion and the Condition Number, the best results are achieved with four centre points. For the estimation pure error and measure of model lack-of-fit, $n_0 = 3$ to 4 centre points are recommended for the composite mixed resolution designs in the spherical regions. Since the replication of the cube and star portions of the designs did not improve the *D*- and *G*-efficiencies and Condition Number, then replication of the cube and star portions is not recommended for the assessment of the CMRD using the *D*- and *G*-efficiencies and the Condition Number in spherical regions. On the other hand, with the advantage of smaller number of design runs, replicating the star portion is recommended over replicating the cube portion in assessing the composite mixed resolution designs using the *I*-optimality criterion.

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Table 2: Summary Statistics for the CMR Designs with Star Replication

Design	α		S_r	G-efficiency					D-efficiency					I-optimality					Condition number				
				$n_0=0$	$n_0=1$	$n_0=2$	$n_0=3$	$n_0=4$	$n_0=0$	$n_0=1$	$n_0=2$	$n_0=3$	$n_0=4$	$n_0=0$	$n_0=1$	$n_0=2$	$n_0=3$	$n_0=4$	$n_0=0$	$n_0=1$	$n_0=2$	$n_0=3$	$n_0=4$
1	S_α	2	1	**90.00**	**91.43**	**90.28**	**88.18**	**85.71**	**76.40**	**75.16**	**73.42**	**71.49**	**69.52**	6.49	4.92	4.14	3.67	3.36	**53.11**	**35.60**	**26.85**	**21.61**	**18.11**
			2	88.89	87.09	84.74	82.24	79.73	74.49	73.35	71.90	70.33	68.70	5.38	4.36	3.77	3.39	3.13	54.23	39.57	31.19	25.77	21.98
			3	77.92	76.87	75.29	73.51	71.67	70.60	69.75	68.64	67.40	66.11	**4.98**	**4.12**	**3.60**	**3.26**	**3.01**	61.36	46.12	36.98	30.89	26.53
	P_α	1.414	1	**100.00**	57.14	**92.84**	**88.80**	**85.11**	**18.16**	**57.63**	**58.15**	**57.45**	**56.34**	2.7E+07	9.88	5.97	4.67	4.01	3.1E+08	**91.01**	**46.03**	**31.04**	**23.56**
			2	85.71	**85.33**	82.049	79.01	76.19	17.41	53.46	54.33	54.04	53.33	1.6E+07	9.68	5.77	4.47	3.81	2.2E+08	109.00	55.02	37.03	28.05
			3	74.999	41.38	73.44	71.07	68.85	16.32	49.40	50.47	50.45	50.02	**1.3E+07**	**9.57**	**5.66**	**4.36**	**3.71**	**2.0E+08**	126.99	64.01	43.03	32.54
2	S_α	2.236	1	**94.55**	**91.90**	**88.64**	**85.39**	**82.29**	**108.96**	**107.8**	**105.61**	**103.00**	**100.29**	18.50	12.57	10.22	8.97	8.18	**110.51**	**62.77**	43.91	**33.82**	**27.53**
			2	77.14	75.89	74.01	71.99	69.98	102.84	102.05	100.52	98.67	96.69	15.26	11.27	9.42	8.35	7.66	119.01	75.54	**55.38**	43.75	36.18
			3	64.94	64.36	63.20	61.86	60.47	95.216	94.83	93.8329	92.5446	91.11	**14.10**	**10.71**	**9.04**	**8.05**	**7.39**	138.18	91.28	68.19	54.45	45.34
	P_α	1.495	1	**91.26**	**88.09**	**84.53**	**81.19**	**78.09**	**76.66**	**79.81**	**79.11**	**77.59**	**75.79**	50.48	16.63	11.92	10.05	9.04	339.69	**83.49**	**47.88**	**33.71**	**26.10**
			2	73.87	72.30	70.10	67.93	65.86	70.71	72.65	72.35	71.42	70.23	30.47	14.11	10.58	9.04	8.17	243.98	87.20	53.34	38.56	30.28
			3	62.14	61.37	59.93	58.43	56.96	64.53	66.05	66.00	65.43	64.63	**23.70**	**12.70**	**9.79**	**8.45**	**7.67**	**221.17**	93.13	59.22	43.54	34.51
3	S_α	2.449	1	**98.22**	**98.79**	**96.74**	**94.65**	**92.60**	**87.76**	**87.63**	**86.79**	**85.68**	**84.44**	32.67	21.31	16.88	14.52	13.06	223.43	**125.49**	**87.32**	**67.00**	**54.38**
			2	86.42	85.46	84.15	82.73	81.29	84.93	84.63	83.93	83.04	82.05	24.65	18.25	15.14	13.31	12.11	**198.33**	129.55	96.24	76.585	63.62
			3	75.65	75.10	74.21	73.20	72.15	80.12	79.89	79.37	78.69	77.92	**21.85**	**16.90**	**14.30**	**12.69**	**11.59**	206.65	142.74	109.1	88.26	74.14
	P_α	1.565	1	**97.71**	**95.52**	**93.308**	**91.17**	**89.11**	**67.44**	**67.52**	**66.95**	**66.13**	**65.19**	34.35	22	17.62	15.38	14.01	208.50	109.45	74.32	56.33	45.40
			2	83.07	81.59	80.06	78.56	77.09	64.06	63.77	63.21	62.52	61.77	20.95	16.48	14.23	12.87	11.97	127.93	85.846	64.67	51.93	43.42
			3	72.37	71.30	70.19	69.08	67.98	59.64	59.31	58.83	58.27	57.66	**16.36**	**13.89**	**12.45**	**11.51**	**10.84**	**102.80**	**76.04**	**60.39**	**50.12**	**42.87**
4	S_α	2.646	1	**99.49**	**104.32**	**94.77**	**92.64**	**90.58**	**111.28**	**108.43**	**106.34**	**104.28**	**102.26**	27.46	27.46	25.79	24.52	23.53	**42.54**	**42.54**	**36.74**	**32.35**	**28.90**
			2	87.54	85.93	84.32	82.72	81.16	107.08	105.26	103.46	101.68	99.94	26.16	24.69	23.56	22.65	21.90	53.09	46.50	41.38	37.27	33.91
			3	78.13	76.91	75.67	74.44	73.22	101.45	99.94	98.43	96.95	95.49	**24.77**	**23.55**	**22.58**	**21.78**	**21.12**	60.99	54.15	48.69	44.24	40.53
	P_α	1.627	1	**96.99**	71.50	**92.65**	**90.39**	**88.25**	**75.95**	**82.42**	**82.51**	**81.77**	**80.73**	843.77	73.35	46.43	37.03	32.24	2.8E+03	186.20	**96.69**	**65.45**	**49.56**
			2	84.969	**80.33**	82.06	80.33	78.66	72.42	77.24	77.51	77.04	76.27	477.04	68.723	44.072	35.18	30.60	1.8E+03	**68.723**	107.50	73.37	55.78
			3	75.62	59.81	73.65	72.29	70.96	67.86	71.82	72.20	71.91	71.36	**354.60**	**65.65**	**42.62**	**34.11**	**29.67**	**1.5E+03**	218.66	118.38	81.31	62.00
5	S_α	2.646	1	**96.84**	**94.95**	**92.90**	**90.95**	**88.99**	**64.23**	**63.29**	**62.25**	**61.16**	**60.07**	43.74	36.03	31.71	28.94	27.02	120.76	**86.80**	**67.83**	**55.67**	**47.23**
			2	82.87	81.63	80.28	78.90	77.53	60.06	59.27	58.43	57.57	56.70	36.07	31.50	28.57	26.54	25.04	**118.06**	92.22	75.68	64.18	55.73
			3	72.24	71.39	70.43	69.43	68.42	55.42	54.81	54.15	53.48	52.79	**33.27**	**29.60**	**27.14**	**25.37**	**24.04**	130.16	104.68	87.55	75.25	65.99
	P_α	1.627	1	**94.37**	**92.26**	**90.12**	**88.05**	**86.07**	**50.12**	**50.08**	**49.53**	**48.82**	**48.05**	88.83	50.84	39.74	34.45	31.35	280.82	127.24	82.40	61.00	48.48
			2	79.98	78.58	77.15	75.64	74.22	45.18	45.88	45.76	44.86	44.26	54.41	39.72	**29.92**	29.92	27.69	175.68	105.69	**59.01**	59.01	**48.39**
			3	69.48	68.48	67.42	66.34	65.28	42.09	41.84	41.47	41.04	40.57	**42.68**	**34.24**	29.94	**27.34**	**25.60**	**143.40**	**96.85**	73.19	**58.88**	49.28
6	S_α	2.646	1	75.62	79.23	80.07	80.04	79.65	84.51	**84.89**	**84.71**	**84.29**	**83.74**	60.34	37.36	28.62	24.02	21.18	470.23	259.18	178.96	**136.70**	**110.61**
			2	**94.96**	**94.29**	**93.42**	**92.48**	**91.52**	**84.68**	84.71	84.46	84.05	83.55	41.23	30.27	24.80	21.53	19.35	349.18	232.38	**174.18**	139.31	116.10
			3	86.35	85.90	85.25	84.53	83.78	82.15	82.10	81.86	81.51	81.08	**34.69**	**27.09**	**22.87**	**20.19**	**18.32**	**324.01**	**231.64**	180.28	147.59	124.94
	P_α	1.627	1	82.86	86.70	88.31	88.90	**88.96**	**66.38**	**66.26**	**65.94**	**65.51**	**65.01**	39.33	30.69	26.24	23.53	21.70	232.93	158.45	120.11	96.74	81.01
			2	**92.13**	**91.10**	**90.08**	**89.07**	88.07	65.61	65.26	64.85	64.40	63.92	24.13	21.44	19.64	18.35	17.37	129.69	104.22	87.13	74.88	65.66
			3	83.20	82.37	81.55	80.74	79.94	62.95	62.58	62.18	61.76	61.33	**18.94**	**17.61**	**16.61**	**15.84**	**15.22**	**95.82**	**81.95**	71.60	**63.59**	**57.20**
7	S_α	2.828	1	**100.00**	**97.53**	**95.16**	**92.89**	**90.70**	**87.27**	**85.18**	**83.19**	**81.28**	**79.45**	46.76	45.60	44.61	43.75	43.00	20.52	18.96	17.62	**16.47**	**15.46**
			2	91.43	89.42	87.48	85.61	83.81	84.96	83.11	81.33	79.63	77.99	44.04	43.17	42.40	41.72	41.11	26.09	24.40	**22.92**	21.62	20.45
			3	83.92	82.26	80.65	79.09	77.58	81.21	79.61	78.06	76.57	75.14	**42.90**	**42.11**	**41.41**	**40.79**	**40.23**	**32.99**	**30.99**	29.22	27.64	26.22
	P_α	1.682	1	**98.31**	**96.23**	**93.82**	**91.46**	**89.21**	**67.23**	**68.19**	**67.73**	**66.87**	**65.85**	272.05	118.79	85.73	71.30	63.22	303.88	108.40	66.24	**47.84**	**37.54**
			2	89.40	87.93	86.06	84.16	82.31	64.68	64.99	64.52	63.77	62.89	179.72	103.18	79.13	67.36	60.38	208.24	100.01	**66.01**	49.37	39.50
			3	81.92	80.85	79.36	77.82	76.28	61.34	61.47	61.05	60.41	59.66	**148.69**	**95.22**	**75.40**	**65.07**	**58.72**	**182.43**	**99.04**	68.12	52.01	42.12
8	S_α	2.828	1	**97.17**	**94.93**	**92.75**	**90.64**	**88.61**	**68.02**	**66.53**	**65.09**	**63.71**	**62.36**	58.08	55.31	53.14	51.40	49.97	**40.89**	**35.91**	**32.01**	**28.88**	**26.31**
			2	85.56	83.90	82.26	80.67	79.12	64.16	62.93	61.74	60.58	59.46	52.93	51.03	49.48	48.18	47.09	46.99	42.35	38.55	35.38	32.69
			3	76.19	74.91	73.65	72.41	71.19	59.75	58.74	57.77	56.81	55.88	**50.85**	**49.22**	**47.86**	**46.70**	**45.71**	56.51	51.38	47.12	43.50	40.41
	P_α	1.682	1	**95.16**	**78.21**	**90.88**	**88.67**	**86.56**	**47.97**	**52.88**	**52.63**	**51.97**	**51.17**	7.3E+03	158.48	99.6981	79.89	69.94	**1.2E+04**	**198.24**	**100.28**	**67.27**	**50.70**
			2	83.22	68.61	80.36	78.65	77.02	44.66	48.59	48.51	48.05	47.46	4.1E+03	153.99	96.5876	77.09	67.26	7.9E+03	225.20	114.56	76.98	58.05
			3	73.96	61.11	72.02	70.67	69.36	41.21	44.59	44.62	44.31	43.86	**3.1E+03**	**151.18**	**94.8149**	**75.55**	**65.82**	6.7E+03	252.18	128.84	86.68	65.40
9	S_α	2.828	1	**92.11**	**95.91**	**96.45**	**96.13**	**95.47**	**45.71**	**45.75**	**45.53**	**45.21**	**44.84**	146.94	84.57	63.81	53.43	47.21	674.74	337.13	**224.77**	**168.62**	**134.94**
			2	89.84	89.23	88.42	87.55	86.66	43.85	43.78	43.58	43.31	43.01	101.23	70.24	56.48	48.70	43.71	516.59	**317.74**	229.47	179.61	147.56
			3	80.41	80.01	79.43	78.77	78.10	41.27	41.19	41.02	40.81	40.56	**85.59**	**63.63**	**52.65**	**46.08**	**41.69**	**489.43**	326.14	244.59	195.68	163.09
	P_α	1.682	1	**99.59**	**98.33**	**97.08**	**95.86**	**94.68**	**37.20**	**36.92**	**36.60**	**36.26**	**35.92**	64.69	56.37	51.24	47.76	45.25	174.05	132.87	107.47	90.25	77.80
			2	87.03	86.08	85.14	84.22	83.32	35.24	34.95	34.65	34.35	34.05	43.04	40.60	38.75	37.31	36.15	96.68	83.29	73.16	65.24	58.88
			3	77.37	76.63	75.90	75.18	74.47	32.84	32.59	32.33	32.07	31.82	**35.54**	**34.36**	**33.39**	**32.59**	**31.91**	**71.30**	**64.17**	**58.34**	**53.49**	**49.39**

NOTE: S_α is the value of Spherical axial distance and P_α is the value of Practical axial distance. The values in bold represents the best results under each criterion

Table 3: Summary Statistics for the CMRDs with Cube Replications

Design	α		C_r	G-efficiency					D-efficiency					I-optimality					Condition number				
				$n_0 = 0$	$n_0 = 1$	$n_0 = 2$	$n_0 = 3$	$n_0 = 4$	$n_0 = 0$	$n_0 = 1$	$n_0 = 2$	$n_0 = 3$	$n_0 = 4$	$n_0 = 0$	$n_0 = 1$	$n_0 = 2$	$n_0 = 3$	$n_0 = 4$	$n_0 = 0$	$n_0 = 1$	$n_0 = 2$	$n_0 = 3$	$n_0 = 4$
1	S_α	2	1	**90.00**	**91.43**	**90.28**	**88.18**	**85.71**	**76.40**	**75.16**	**73.42**	**71.49**	**69.52**	6.49	4.92	4.14	3.67	3.36	**53.11**	**35.60**	**26.85**	**21.61**	18.11
			2	55.56	58.48	59.74	60.12	60.00	71.78	71.64	71.11	70.37	69.51	4.29	3.38	2.87	2.53	3.13	66.05	48.23	38.05	31.47	**2.30**
			3	40.38	43.16	44.76	45.68	46.15	67.98	68.25	68.19	67.93	67.55	**3.54**	**2.82**	**2.40**	**2.11**	**1.91**	82.36	61.97	49.74	41.58	35.76
	P_α	1.414	1	**100.00**	**57.14**	**92.84**	88.80	85.11	**18.16**	**57.63**	**58.15**	57.45	56.34	2.7E+07	9.88	5.97	4.67	4.01	**3.1E+08**	**91.01**	**46.03**	**31.04**	**23.56**
			2	60.00	32.43	63.16	**92.31**	**90.01**	17.27	55.53	57.15	**57.52**	**57.39**	2.4E+07	8.98	5.07	3.76	3.11	5.0E+08	163.00	82.01	55.02	41.53
			3	42.86	22.64	44.44	65.45	69.65	16.44	53.13	55.12	55.90	56.18	**2.3E+07**	**8.66**	**4.75**	**3.45**	**2.79**	7.0E+08	235.00	118.01	79.02	59.52
2	S_α	2.236	1	**94.55**	**91.90**	**88.64**	**85.39**	**82.29**	**108.96**	107.84	105.61	103.00	100.29	18.50	12.57	10.22	8.97	8.18	**110.51**	**62.77**	**43.92**	**33.82**	**27.53**
			2	76.52	79.68	80.27	79.84	78.95	113.28	113.41	112.65	111.47	110.07	12.30	8.79	7.15	6.20	5.58	130.27	83.14	61.12	48.37	40.05
			3	56.86	60.09	61.34	61.72	61.65	113.13	**113.83**	**113.75**	**113.27**	**112.55**	**10.18**	**7.38**	**5.99**	**5.16**	**4.60**	158.20	105.28	78.96	63.21	52.73
	P_α	1.495	1	**91.26**	**88.09**	84.53	81.19	78.09	**76.66**	79.81	79.11	77.59	75.79	50.48	16.63	11.92	10.05	9.04	**339.69**	**83.49**	**47.88**	**33.71**	**26.10**
			2	84.39	62.46	**98.95**	**96.57**	**94.28**	81.12	85.67	86.35	86.05	85.33	45.06	14.16	9.52	7.65	6.64	551.60	144.55	83.43	58.77	45.44
			3	61.43	44.67	76.01	78.24	77.63	81.61	**86.73**	**88.03**	**88.34**	**88.18**	**43.22**	**13.29**	**8.68**	**6.81**	**5.79**	768.05	205.81	119.08	83.90	64.85
3	S_α	2.449	1	**98.22**	**98.79**	**96.74**	**94.65**	**92.60**	**87.76**	**87.63**	**86.79**	**85.68**	84.44	32.67	21.31	16.88	14.52	13.06	**223.43**	**125.49**	**87.32**	**67.00**	**54.38**
			2	59.98	63.33	64.47	64.76	64.67	85.25	85.77	85.74	85.44	**84.98**	24.09	16.11	12.62	10.66	9.41	311.16	189.66	136.47	106.62	87.52
			3	43.27	46.12	47.33	47.87	48.08	82.26	83.01	83.28	83.30	83.17	**21.17**	**14.24**	**11.07**	**9.26**	**8.08**	407.23	255.47	186.18	146.50	120.79
	P_α	1.565	1	**97.71**	**95.52**	**93.31**	**91.17**	**89.11**	**67.44**	**67.52**	66.95	66.13	65.19	34.35	22.00	17.62	15.38	14.01	**208.50**	**109.45**	**74.32**	**56.33**	**45.40**
			2	64.83	71.35	73.49	74.21	74.29	66.34	67.09	**67.20**	**67.04**	**66.73**	30.39	18.58	14.27	12.04	10.67	373.53	199.93	136.60	103.81	83.75
			3	45.85	50.84	52.68	53.47	53.78	64.31	65.30	65.67	65.78	65.73	**29.03**	**17.40**	**13.12**	**10.89**	**9.52**	539.42	290.62	199.00	151.36	122.16
4	S_α	2.646	1	**99.13**	**96.94**	**94.77**	**92.64**	**90.58**	110.51	108.43	106.34	104.28	102.26	29.76	27.46	25.79	24.52	23.53	**50.52**	**42.54**	6.74	**32.35**	**28.90**
			2	62.98	63.57	63.87	63.96	63.90	**111.28**	**110.30**	**109.28**	108.23	107.17	18.09	16.85	15.88	15.10	14.46	61.24	53.70	**47.83**	43.12	39.27
			3	45.22	45.91	46.41	46.75	46.98	110.08	109.53	108.94	**108.31**	**107.67**	**14.10**	**13.15**	**12.38**	**11.76**	**11.24**	75.47	67.10	60.42	54.95	50.40
	P_α	1.627	1	**96.99**	**69.71**	**92.65**	90.39	88.25	75.95	82.42	82.51	81.77	80.73	843.77	73.35	46.43	37.03	32.24	**2750.00**	**186.20**	**96.69**	**65.45**	**49.56**
			2	69.01	39.45	75.02	**92.08**	**91.07**	**77.32**	**84.70**	85.75	85.91	85.71	788.33	65.64	38.73	29.31	24.51	4780.00	343.73	178.60	120.79	91.33
			3	48.45	27.24	51.98	66.69	66.24	76.82	84.45	**85.86**	**86.39**	**86.54**	**769.79**	**63.01**	**36.11**	**26.67**	**21.87**	6830.00	501.30	260.52	176.13	133.12
5	S_α	2.646	1	**96.84**	**94.95**	**92.95**	**90.95**	**88.99**	**64.23**	**63.29**	**62.25**	**61.16**	**60.07**	43.74	36.03	31.71	28.94	27.02	**120.76**	**86.84**	**67.83**	**55.67**	**47.23**
			2	70.65	72.29	73.00	73.20	73.11	60.08	59.73	59.30	58.81	58.30	29.01	24.25	21.29	19.29	17.83	155.41	119.00	96.44	81.09	69.97
			3	51.18	52.72	53.60	54.09	54.34	56.64	56.52	56.32	56.07	55.78	**23.97**	**20.06**	**17.55**	**15.81**	**14.52**	196.57	153.82	126.37	107.25	93.17
	P_α	1.627	1	**94.37**	**92.26**	**90.12**	88.05	86.07	**50.12**	**50.08**	**49.53**	**48.82**	**48.05**	88.83	50.84	39.74	34.45	31.35	**280.82**	**127.24**	**82.40**	**61.00**	**48.48**
			2	77.42	76.75	88.99	**89.63**	**89.55**	47.39	47.83	47.80	47.59	47.28	78.45	42.41	31.51	26.25	23.15	497.30	231.31	150.84	111.97	89.08
			3	54.83	53.80	63.96	64.76	65.00	44.88	45.47	45.62	45.60	45.49	**74.91**	**39.53**	**28.69**	**23.44**	**20.34**	715.16	335.62	219.38	163.02	129.74
6	S_α	2.646	1	**75.62**	**79.23**	**80.07**	**80.04**	**79.65**	**84.51**	**84.89**	**84.71**	**84.29**	**83.74**	60.34	37.36	28.62	24.02	21.18	**470.23**	**259.18**	**178.96**	**136.70**	**110.61**
			2	43.53	46.28	47.32	47.76	47.92	79.99	80.71	80.98	81.03	80.96	49.00	30.57	23.02	18.90	16.32	746.47	434.20	306.20	236.53	192.71
			3	30.59	32.71	33.60	34.04	34.27	76.43	77.26	77.67	77.89	77.98	**45.17**	**28.19**	**21.04**	**17.10**	**14.61**	1.0E+03	610.55	433.87	336.54	274.90
	P_α	1.627	1	**82.86**	**86.70**	**88.31**	**88.90**	**88.96**	**66.38**	**66.26**	**65.94**	**65.51**	**65.01**	39.33	30.69	26.24	23.53	21.70	**232.93**	**158.45**	**120.11**	**96.74**	**81.01**
			2	46.00	48.55	50.91	50.54	50.92	63.35	63.61	64.09	63.64	63.53	34.71	26.23	**19.12**	19.12	17.30	440.62	301.62	185.03	185.03	155.10
			3	31.82	33.70	34.69	35.27	35.63	60.72	60.42	61.31	61.40	61.43	**33.14**	**24.72**	20.33	**17.63**	**15.80**	648.57	444.91	338.64	273.38	229.22
7	S_α	2.828	1	**100.00**	**97.53**	**95.16**	**92.89**	**90.70**	**87.27**	**85.18**	**83.19**	**81.28**	79.45	46.76	45.60	44.61	43.75	43.00	**20.52**	**18.96**	**17.62**	**16.47**	**15.46**
			2	58.83	58.62	58.36	58.07	57.75	83.80	82.76	81.75	80.75	**79.78**	25.56	25.08	24.65	24.25	23.90	20.74	19.67	18.71	17.84	17.05
			3	42.00	42.07	42.12	42.13	42.11	80.97	80.31	79.66	79.02	78.38	**18.36**	**18.04**	**17.75**	**17.48**	**17.23**	23.49	22.47	21.54	20.69	19.90
	P_α	1.682	1	**98.31**	**96.23**	**93.82**	91.46	89.21	**67.23**	**68.19**	**67.73**	66.87	65.85	272.05	118.79	85.73	71.30	63.22	**303.88**	**108.40**	**66.24**	**47.84**	**37.54**
			2	65.29	56.04	89.52	**94.95**	**95.83**	65.10	66.69	67.02	**66.94**	**66.66**	227.95	97.76	67.29	53.69	46.00	508.80	193.56	119.80	86.89	68.25
			3	45.47	38.84	61.99	69.07	70.21	63.11	64.89	65.50	65.70	65.71	**213.17**	**90.51**	**60.96**	**47.67**	**40.12**	716.76	278.84	173.37	125.93	98.97
8	S_α	2.828	1	**97.17**	**94.93**	**92.75**	**90.64**	**88.61**	**68.02**	**66.53**	**65.09**	**63.71**	**62.36**	58.08	55.31	53.14	51.40	49.97	**40.89**	**35.91**	**32.01**	**28.88**	**26.31**
			2	71.43	71.51	71.43	71.24	70.95	63.68	62.96	62.25	61.55	60.85	33.40	32.05	30.92	29.97	29.16	45.54	41.43	38.01	35.11	32.63
			3	51.47	51.81	52.04	52.18	52.25	60.32	59.88	59.44	59.00	58.56	**24.96**	**23.98**	**23.15**	**22.43**	**21.80**	53.92	49.65	46.02	42.88	40.15
	P_α	1.682	1	**95.16**	**78.21**	**90.88**	88.67	86.56	**47.97**	**52.88**	**52.63**	**51.97**	**51.17**	7.3E+03	158.48	99.70	79.89	69.94	**1.2E+04**	**198.24**	**100.28**	**67.27**	**50.70**
			2	79.27	43.01	84.08	**98.58**	**97.25**	45.34	50.46	50.79	50.70	50.45	6.8E+03	140.48	81.62	61.77	51.80	2.1E+04	365.63	184.78	123.78	93.15
			3	55.72	29.66	58.22	77.03	76.35	43.11	48.15	48.67	48.79	48.75	**6.6E+03**	**134.35**	**75.47**	**55.60**	**45.62**	2.9E+04	533.04	269.29	180.30	135.60
9	S_α	2.828	1	**92.11**	**95.91**	**96.45**	**96.13**	**95.47**	**45.71**	**45.75**	**45.53**	**45.21**	**44.84**	146.94	84.57	63.81	53.43	47.21	674.74	**337.13**	**224.77**	**168.62**	**134.94**
			2	54.00	57.05	57.99	58.30	58.34	40.36	40.58	40.60	40.54	40.44	118.47	68.72	50.65	41.32	35.61	**1.1E+03**	558.75	380.91	288.98	232.82
			3	38.24	40.63	41.48	41.85	42.00	36.92	37.19	37.29	37.31	37.29	**108.83**	**63.12**	**46.00**	**37.04**	**31.53**	1.4E+03	782.08	537.56	409.56	330.82
	P_α	1.682	1	**99.59**	**98.33**	**97.08**	**95.86**	**94.68**	**37.20**	**36.92**	**36.60**	**36.26**	**35.92**	**37.20**	56.37	51.24	47.76	45.25	**174.05**	**132.87**	107.47	**90.25**	**77.80**
			2	57.56	59.32	60.33	60.91	61.23	33.10	33.04	32.95	32.83	32.70	53.59	45.41	40.32	36.85	34.33	329.70	252.81	205.02	172.45	148.82
			3	40.03	41.37	42.18	42.70	43.03	30.36	30.38	30.36	30.31	30.26	49.83	**41.68**	**36.61**	**33.14**	**30.62**	485.55	372.86	**36.61**	254.71	219.89