Effect of power transformations on a Weibull-distributed error component of a multiplicative error model

Chris Uchechi Onyemachi*[1], Johnson Ohakwe[2] and Ifeanyi Sidney Onyeagu[1]

[1]*Department of Statistics, Nnamdi Azikiwe University, Awka*
[2]*Department of Mathematics and Statistics, Federal University Otuoke*

The error component of a Multiplicative Error Model (MEM) can possibly be a Weibull distribution ($W(\sigma,n)$; σ and n are shape and scale parameters respectively). Data transformation is a popular remedial measure to stabilize the variance of a data set prior to statistical modeling. Therefore, in this paper the effects of power transformations on the mean and variance of a Weibull distributed error component of a MEM are investigated. The popular transformations - inverse, square-root, inverse-square-root, square, inverse-square, cube root, inverse cube root, cube and inverse cube transformations were studied. The probability density function (pdf) and the k^{th} raw moment of the p-th power–transformed Weibull random variable are obtained. The mean and variance of $W(\sigma,n)$ and those of the power – transformed distributions are calculated for σ = 6, 7, . . ., 99, 100 with the corresponding values of n for which the mean of the untransformed distribution is equal to one. The relative changes in mean and variance are used for the investigations. For all the transformations, the means of the power transformed distributions are not different from 1. For variances, it was found that there are relative increases for the inverse, square, inverse square, cube and inverse cube transformations. However, the square-root, inverse square root, cube root and inverse-cube-root transformations decreased the variance relative to the variance of the untransformed distribution. This paper concludes that the square-root, inverse square root, cube root and inverse-cube-root transformations would yield better results as they reveal constancy in variance when using MEM with a Weibull distributed error component and where data transformation is deemed necessary to stabilize the variance of the data set.

Keywords: multiplicative error model; Weibull distribution; power transformation; mean; variance

1 Introduction

Multiplicative Error Models (MEMs) were originally introduced as Autoregressive Conditional Duration (ACD) models by Engle and Russell (1998) and were generalized to any non-negative time varying event process by Engle (2002). MEMs provide an observation-driven approach for dynamic non-negative variables; see Russell and Engle (2010). Let $\{x_t\}$ be a discrete time process defined on $[0,+\infty)$, $t \in \mathbf{N}$, where $\mathbf{N}$ is the set of N natural numbers and let ψ_{t-1} be the information available for forecasting x_t. Brownlees et al. (2011) showed that $\{x_t\}$ follows a MEM if it can be expressed as

$$x_t = \mu_t \varepsilon_t , \tag{1}$$

where conditionally on ψ_{t-1}, μ_t is a positive quantity that evolves deterministically according to a parameter vector θ.

(2)

*Corresponding Author; E-mail: ucheonyemachi@yahoo.com

ε_t is a random variable with probability density function (pdf) defined over $[0,+\infty)$ support, unit mean and unknown constant variance.

$$\varepsilon_t \mid \psi_{t-1} \sim D^{+}(1,\sigma^2). \tag{3}$$

Irrespective of the specification of the function $\mu(.)$ and of the distribution D^{+} (any distribution) equations (1), (2) and (3), according to Engle (2002), must evolve

$$E(x_t \mid \psi_{t-1}) = \mu_t, \tag{4}$$

$$V(x_t \mid \psi_{t-1}) = \sigma^2 \mu_t^2. \tag{5}$$

The assumption, according to Engle and Russell (1998), is that the time dependence in the durations can be subsumed in their conditional expectations (4), in such a way that $x_t \mid \psi_t$ is independently and identically distributed, where ψ_{t-1} denotes the information set available at time, $t-1$.

No doubt the realization of (3) supports such distributions as Weibull, Gamma, Log Normal Andersen, Bollerslev, Diebold and Ebens (2001), Bandi and Russell (2006), Panton (2010), Manganelli (2005). In this case, ε_t are independently and identically distributed as a Weibull $(1,\alpha)$ probability distribution, as ψ_t are proportional to the conditional expectation of x_t as explained below

$$\psi_t = \alpha x_{t-1} + \beta \psi_{t-1} \tag{6}$$

with the constraints on the coefficients as follows: $\beta \geq 0, \alpha \geq 0$ and $\alpha + \beta < 1$. The conditional lag in time is guaranteed by the first three conditions while the last condition ensures non-negativity.
The property (4) provides us with a link on (6) which gives

$$\psi_t = \Gamma\left(1 + \frac{1}{\alpha}\right)\mu_t \tag{7}$$

where $\Gamma(.)$ is the gamma function. If $\alpha = 1$, the Weibull distribution becomes an exponential one. Here, $\mu_t = \psi_t$. The property of (3) also in realization support the left truncated normal distribution whose effect on the error component of the multiplicative time series has been studied via the logarithm, inverse, square, inverse square, square root transformations Iwueze (2007), Nwosu et al. (2013), Ohakwe et al. (2013) and Dike et al. (2016).

2 Motivation

Most statistical tests and procedures contain assumptions about the data to which they will be applied, usually such as that the data are normally distributed, variance components are additive, multiplicative etc. When data fail to satisfy such criteria, it implies that the results of a statistical procedure or test has been violated and apparently tend to be biased. Data that do not satisfy statistical assumptions may often be transformed mathematically into a form that permits standard statistical tests to perform adequately.

Unitized power transformation to maximize data variance which is achievable with transformations in simple functional forms and also a class of multiplicative error models is the focus of this paper. In the case of MEM, the basic assumptions of the error component is a unit mean and constant variance σ^2. The essence of transformation is to stabilize the variance and at same time examine its impact on the unit mean assumption. The above motivation for transforming MEM data are not the only one that endow optimal characteristics to the resulting variables. Another reason is to increase the amount of information revealed in the data Stanley (2006).

Power transformation such as $y = x^{\lambda}$ (Moore 1957; Healy and Taylor 1962) is used to stabilize the error in data and creates new variable that is "homoscedastic" (has constant error variance across the range of the new transformed variable), and this attribute can significantly simplify subsequent statistical tests, numerical procedures and the graphical presentation of the data. Evaluation of homoscedastic data is far simpler and leads to more compelling conclusions because the data are evaluated on an "even playing field". A random variable X has a Weibull distribution $X \sim Weibull(\alpha, n)$, with shape parameters $(\alpha > 0)$, and scale parameter $(n > 0)$. With the probability density function,

$$\mathbf{f}(\mathbf{x}) = \left(\frac{\mathbf{n}}{\boldsymbol{\alpha}}\right)\left(\frac{\mathbf{x}}{\boldsymbol{\alpha}}\right)^{\sigma - 1} \exp\left\{-\left(\frac{\mathbf{x}}{\boldsymbol{\alpha}}\right)^{\alpha}\right\}, \mathbf{x} > \mathbf{0}. \qquad (8)$$

The Weibull distribution is used in reliability and survival analysis to model the lifetime of objects, organisms and service time. The baseline Weibull distribution depicts same identity with the accelerated life and Cox proportional hazards model (Adejumo et al., 2016).

3 Data Transformation

To enhance forecasting, data transformation appears to be the most frequent reason for researchers to make the distribution of their data "normal" and thus fulfill one of the assumptions of conducting a parametric means comparison. Other reasons for data transformation include more informative graphs of the data, better outlier identification and increasing the sensitivity of statistical tests. Succinctly put, data transformation is a mathematical operation that changes the measurement scale of a variable. According to Chartfield (2004), if there is trend in the series and the variance appears to increase with the mean, then it may be advisable to transform the data; and in particular – if the standard deviation is directly proportional to the mean, a logarithmic transformation is appropriate. He proceeded to outline the following reasons for transformation i). variance stabilization ii). to make the seasonal effect additive and iii) to normalize the data. Bartlett (1947), Turkey (1957), and Osborne (2002) made specific conditions and recommendations on choice and caution in data transformations. These works by Dike et al. (2016) did not consider the implications of transformations on components of the multiplicative time series model until Iwueze (2007), pioneered a work on the implications of logarithmic transformations on the error component of the multiplicative model. Interestingly, the work elicited spontaneous interest in this area of time series analysis, thus adding to the litany of several works including (Akpanta, 2007) and (Iwueze, Akpanta, and Iwu, 2009). The popular series of transformations range from adding constants to multiplying, squaring or raising to a power, converting to logarithmic scale, inverting and reflecting, taking square root of the values, and even applying trigonometric transformations such as sine wave transformations. The development in transformations, are interventions and remedy for outliers, failures on normality, linearity and homoscedasticity. Some common power transformations on a variable X include X_t^2, $\sqrt{X_t}$, $\frac{1}{X_t}$, $\frac{1}{\sqrt{X_t}}$, $\frac{1}{X_t^2}$ and $\log_e X_t$. For further details on transformation, see Bartlett (1947); Box and Cox (1964); Akpanta and Iwueze (2009); Otuonye et al. (2011) and Ibe et al. (2013).

4 The Weibull *P*-th Transformed Random Variable

Power transformation is a form of transformation that is frequently used in statistical analysis and is defined as follows (Ozdemir, 2017)

$$\mathbf{Y} = \begin{cases} \mathbf{X^p}, & \mathbf{p \neq 0} \\ \mathbf{\log(X)}, & \mathbf{p = 0} \end{cases} \tag{9}$$

where Y is the transformed variable, X, the untransformed variable and *p*, a constant. In this study we shall consider when $p \neq 0$. Our choice of *p*-th power transformation is based on the fact that the commonly used power transformations in the literature are subclasses of it. The popular transformations for various values of *p* are shown in Table 1.

Table 1: The Popular Transformations for Various Values of *P*

S/N	*p*	Type of Transformation
1	1	No transformation (NT)
2	-1	Inverse transformation (IT)
3	½	Square root transformation (SRT)
4	- ½	Inverse square root transformation (ISRT)
5	2	Square transformation (ST)
6	-2	Inverse square transformation (IST)
7	1/3	Cube root transformation (CRT)
8	-1/3	Inverse cube root transformation (ICRT)
9	3	Cube transformation (CT)
10	-3	Inverse cube transformation (ICT)

Suppose $Y_t = X_t^{\mathbf{p}}$,

$$X_t = Y_t^{\frac{1}{p}} \tag{10}$$

and

$$|J| = \left|\frac{d\,x_t}{d\,y_t}\right| = \left|\frac{1}{p}\, y_t^{\frac{1}{p}-1}\right| = \left|\frac{1}{p}\right| y_t^{\frac{1}{p}-1}, \tag{11}$$

where $|J|$ is the absolute value of the Jacobian of the p-th power transformation. The pdf of Y_t, denoted as $f(y_t)$ is then obtained as

$$f(y_t) = f\left(x_t = y_t^{\frac{1}{p}}\right)\left|\frac{d\,x_t}{d\,y_t}\right| \quad \text{(Ramachandran and Tsokos, 2009).} \tag{12}$$

Now, suppose the error component (e_t) of a Multiplicative Error Model (MEM) is assumed to follow Weibull distribution, then the probability density function (pdf) of e_t, denoted as $f(e_t)$, is given as follows:

$$\mathbf{f}(\mathbf{e_t}) = \left(\frac{\boldsymbol{\sigma}}{\mathbf{n}}\right)\left(\frac{\mathbf{e_t}}{\mathbf{n}}\right)^{\boldsymbol{\sigma}-1} \exp\left\{-\left(\frac{\mathbf{e_t}}{\mathbf{n}}\right)^{\boldsymbol{\sigma}}\right\}, \mathbf{e_t} > \mathbf{0} \tag{13}$$

where σ and n are the shape and scale parameters, respectively. The mean ($E(e_t) = \boldsymbol{\mu}_{\mathbf{e_t}}$) and variance ($\mathbf{Var}(\mathbf{e_t}) = \boldsymbol{\sigma}^2_{\mathbf{e_t}}$) of e_t are given as

$$\mathbf{E}(\mathbf{e_t}) = \boldsymbol{\mu}_{\mathbf{e_t}} = \mathbf{n}\,\boldsymbol{\Gamma}\left(\mathbf{1} + \tfrac{\mathbf{1}}{\boldsymbol{\sigma}}\right) \tag{14}$$

and

$$\boldsymbol{\sigma}^2_{\mathbf{e_t}} = \mathbf{n}^2\,\boldsymbol{\Gamma}\left(\mathbf{1}+\tfrac{\mathbf{2}}{\boldsymbol{\sigma}}\right) - \left[\mathbf{n}\,\boldsymbol{\Gamma}\left(\mathbf{1}+\tfrac{\mathbf{1}}{\boldsymbol{\sigma}}\right)\right]^2. \tag{15}$$

Using (12) and (13), we obtain the pdf of the p-th transformed Weibull distribution denoted by $\mathbf{f}(\mathbf{y}_t)$ as

$$\mathbf{f}(\mathbf{y_t}) = \frac{\sigma}{|\mathbf{p}|}\left(\frac{\mathbf{1}}{\mathbf{n}}\right)^{\sigma} \mathbf{y_t}^{\frac{\sigma}{\mathbf{p}}-\mathbf{1}} \mathbf{exp}\left\{-\left(\frac{\mathbf{y_t}^{\frac{1}{\mathbf{p}}}}{\mathbf{n}}\right)^{\sigma}\right\}, \mathbf{y_t} > \mathbf{0} \tag{16}$$

If p = 1 in (16), we obtain

$$\mathbf{f}(y_t) = \left(\frac{\boldsymbol{\sigma}}{\mathbf{n}}\right)\left(\frac{y_t}{\mathbf{n}}\right)^{\boldsymbol{\sigma}-1} \mathbf{exp}\left\{-\left(\frac{\mathbf{y_t}}{\mathbf{n}}\right)^{\boldsymbol{\sigma}}\right\}, \mathbf{y_t} > \mathbf{0}, \tag{17}$$

which established that no transformation is required when p =1.

To establish that $\mathbf{f}(\mathbf{y}_t)$ is a proper pdf, we have to show that $\int_0^\infty f(y_t)\,dy_t = 1$.

$f(y_t) \geq 0,\ \forall y_t$. We now proceed as follows;

$$\int_0^\infty f(y_t)dy_t = \frac{\sigma}{|\mathbf{p}|}\left(\frac{\mathbf{1}}{\mathbf{n}}\right)\int_0^\infty \mathbf{y}_t^{\frac{\sigma}{\mathbf{p}}-\mathbf{1}} \mathbf{exp}\left\{-\left(\frac{\mathbf{y_t}^{\frac{1}{\mathbf{p}}}}{\mathbf{n}}\right)^{\sigma}\right\} dy_t \tag{18}$$

In (18) if,

$$Z = \left(\frac{y^{\frac{1}{p}}}{n}\right)^{\sigma} \tag{19}$$

we obtain

$$\mathbf{y_t} = \mathbf{n^p}\,\mathbf{z}^{\frac{\mathbf{p}}{\boldsymbol{\sigma}}} \text{ and } \mathbf{dy_t} = \frac{\mathbf{p}\,\mathbf{n}^{\mathbf{p}}\mathbf{z}^{\frac{\mathbf{p}}{\boldsymbol{\sigma}}-\mathbf{1}}}{\boldsymbol{\sigma}}\mathbf{dz} \tag{20}$$

Now substituting (19) and (20) into (18), we have that

$$\int_0^{\infty} f(y_t)\,dy_t = \int_0^{\infty} e^{-Z}\,dz = 1$$

which shows that (16) is a proper pdf.The pdf of the transformed Weibull variable under the various power transformations are given in Table 2.

Table 2: The pdf of the transformed Weibull (Y_t) variable under the various power transformations

S/N	p	Transformation	Probability Density Function (pdf)
1	1	NT	$f(y_t) = \left(\frac{\sigma}{n}\right)\left(\frac{y_t}{n}\right)^{\sigma-1} \exp\left\{-\left(\frac{y_t}{n}\right)^{\sigma}\right\}, y_t > 0$
2	-1	IT	$f(y_t) = \sigma\left(\frac{1}{n}\right)^{\sigma}(y_t)^{-\sigma-1} \exp\left\{-\left(\frac{1}{n y_t}\right)^{\sigma}\right\}, y_t > 0$
3	½	SRT	$f(y_t) = 2\sigma\left(\frac{1}{n}\right)^{\sigma}(y_t)^{2\sigma-1} \exp\left\{-\left(\frac{y_t^2}{n}\right)^{\sigma}\right\}, y_t > 0$
4	- ½	ISRT	$f(y_t) = 2\sigma\left(\frac{1}{n}\right)^{\sigma}(y_t)^{-2\sigma-1} \exp\left\{-\left(\frac{1}{n y_t^2}\right)^{\sigma}\right\}, y_t > 0$
5	2	ST	$f(y_t) = \frac{\sigma}{2}\left(\frac{1}{n}\right)^{\sigma} y^{\frac{\sigma}{2}-1} \exp\left\{-\left(\frac{y^{\frac{1}{2}}}{n}\right)^{\sigma}\right\}, y_t > 0$
6	-2	IST	$f(y_t) = \frac{\sigma}{2}\left(\frac{1}{n}\right)^{\sigma} y^{-\frac{\sigma}{2}-1} \exp\left\{-\left(\frac{1}{n y^{\frac{1}{2}}}\right)^{\sigma}\right\}, y_t > 0$
7	1/3	CRT	$f(y_t) = 3\sigma\left(\frac{1}{n}\right)^{\sigma}(y_t)^{3\sigma-1} \exp\left\{-\left(\frac{y_t^3}{n}\right)^{\sigma}\right\}, y_t > 0$
8	-1/3	ICRT	$f(y_t) = 3\sigma\left(\frac{1}{n}\right)^{\sigma}(y_t)^{-3\sigma-1} \exp\left\{-\left(\frac{1}{n y_t^3}\right)^{\sigma}\right\}, y_t > 0$
9	3	CT	$f(y_t) = \frac{\sigma}{3}\left(\frac{1}{n}\right)^{\sigma} y^{\frac{\sigma}{3}-1} \exp\left\{-\left(\frac{y^{\frac{1}{3}}}{n}\right)^{\sigma}\right\}, y_t > 0$
10	-3	ICT	$f(y_t) = \frac{\sigma}{3}\left(\frac{1}{n}\right)^{\sigma} y^{-\frac{\sigma}{3}-1} \exp\left\{-\left(\frac{1}{n y^{\frac{1}{3}}}\right)^{\sigma}\right\}, y_t > 0$

The K-th Raw Moment of Y_t [$E(Y^k)$]

The K^{th} raw moment of Y_t, denoted as $E\left(Y_t^k\right)$, is obtained as follows:

$$E\left(Y_t^k\right)=\int_0^\infty y_t^k f\left(y_t\right)dy_t = \frac{\sigma}{|p|}\left(\frac{1}{n}\right)^{\sigma}\int_0^\infty y^k\, y^{\frac{\sigma}{p}-1}\exp\left\{-\left(\frac{y^{\frac{1}{p}}}{n}\right)^{\sigma}\right\}dy_t$$

$$=\frac{\sigma}{|p|}\left(\frac{1}{n}\right)^{\sigma}\int_0^\infty y^{\frac{\sigma}{p}+k-1}\exp\left\{-\left(\frac{y^{\frac{1}{p}}}{n}\right)^{\sigma}\right\}dy_t\,. \qquad (21)$$

If we insert the substitutions in (19) and its corresponding results in (20) into (21), we obtain

$$E\left(Y_t^k\right)=\frac{\sigma}{|p|}n^{-\sigma}\int_0^\infty\left(n^p z^{\frac{p}{\sigma}}\right)^{\frac{\sigma}{p}+k-1}\exp\{-z\}\frac{p\,n^p z^{\frac{p}{\sigma}-1}}{\sigma}dz = n^{pk}\int_0^\infty z^{\left(1+\frac{pk}{\sigma}\right)-1}e^{-z}\,dz$$

$$= n^{pk}\,\Gamma\left(1+\frac{pk}{\sigma}\right). \qquad (22)$$

Table 3: Expressions for the Means and Variances for the Various Power Transformations

S/N	P	Type of Transformation	$E(Y_t) = \mu_t$	$\sigma^2_{y_t}$
1	-1	IT	$n^{-1}\Gamma\left(1-\frac{1}{\sigma}\right)$, σ>1	$n^{-2}\Gamma\left(1-\frac{2}{\sigma}\right)-\left[n^{-1}\Gamma\left(1-\frac{1}{\sigma}\right)\right]^2$,σ>2
2	½	SRT	$n^{\frac{1}{2}}\Gamma\left(1+\frac{1}{2\sigma}\right)$	$n\Gamma\left(1+\frac{1}{\sigma}\right)-\left[n^{\frac{1}{2}}\Gamma\left(1+\frac{1}{2\sigma}\right)\right]^2$
3	- ½	ISRT	$n^{-\frac{1}{2}}\Gamma\left(1-\frac{1}{2\sigma}\right)$, σ>1	$n^{-1}\Gamma\left(1-\frac{1}{\sigma}\right)-\left[n^{-\frac{1}{2}}\Gamma\left(1-\frac{1}{2\sigma}\right)\right]^2$ σ>1
4	2	ST	$n^{2}\Gamma\left(1+\frac{2}{\sigma}\right)$	$n^{4}\Gamma\left(1+\frac{4}{\sigma}\right)-\left[n^{2}\Gamma\left(1+\frac{2}{\sigma}\right)\right]^2$
5	-2	IST	$n^{-2}\Gamma\left(1-\frac{2}{\sigma}\right)$, σ>2	$n^{-4}\Gamma\left(1-\frac{4}{\sigma}\right)-\left[n^{-2}\Gamma\left(1-\frac{2}{\sigma}\right)\right]^2$,σ>2
6	1/3	CRT	$n^{\frac{1}{3}}\Gamma\left(1+\frac{1}{3\sigma}\right)$	$n^{\frac{2}{3}}\Gamma\left(1+\frac{2}{3\sigma}\right)-\left[n^{\frac{1}{3}}\Gamma\left(1+\frac{1}{3\sigma}\right)\right]^2$
7	- 1/3	ICRT	$n^{-\frac{1}{3}}\Gamma\left(1-\frac{1}{3\sigma}\right)$	$n^{-\frac{2}{3}}\Gamma\left(1-\frac{2}{3\sigma}\right)-\left[n^{-\frac{1}{3}}\Gamma\left(1-\frac{1}{3\sigma}\right)\right]^2$
8	3	CT	$n^{3}\Gamma\left(1+\frac{3}{\sigma}\right)$	$n^{6}\Gamma\left(1+\frac{6}{\sigma}\right)-\left[n^{3}\Gamma\left(1+\frac{3}{\sigma}\right)\right]^2$
9	-3	ICT	$n^{-3}\Gamma\left(1-\frac{3}{\sigma}\right)$,σ>3	$n^{-6}\Gamma\left(1-\frac{6}{\sigma}\right)-\left[n^{-3}\Gamma\left(1-\frac{3}{\sigma}\right)\right]^2$, σ>6

When k = 1, we obtain

$$E\left(Y_t\right) = \mathbf{n}^{\mathbf{p}}\ \mathbf{\Gamma}\left(\mathbf{1}+\frac{\mathbf{p}}{\boldsymbol{\sigma}}\right), \tag{23}$$

and for k = 2, yields

$$\mathbf{E}\left(\mathbf{Y}_{\mathbf{t}}^{\mathbf{2}}\right) = \mathbf{n}^{\mathbf{2p}}\ \mathbf{\Gamma}\left(\mathbf{1}+\frac{\mathbf{2p}}{\boldsymbol{\sigma}}\right). \tag{24}$$

Thus,

$$\sigma_{y_t}^2 = \mathbf{E}(\mathbf{Y}_{\mathbf{t}}^2) - [\mathbf{E}(\mathbf{Y}_{\mathbf{t}})] = \mathbf{n}^{\mathbf{2p}}\ \mathbf{\Gamma}\left(\mathbf{1}+\frac{\mathbf{2p}}{\boldsymbol{\sigma}}\right) - \left[\mathbf{n}^{\mathbf{p}}\ \mathbf{\Gamma}\left(\mathbf{1}+\frac{\mathbf{p}}{\boldsymbol{\sigma}}\right)\right]^{\mathbf{2}}. \tag{25}$$

The Expressions for the means and variances under the various power transformations are given in Table 3.

3 Relative Change in Means and Variances of the Transformed and Untransformed Distributions

In this section, the means and variances of the transformed and the untransformed distributions would be obtained followed by the computations of the relative changes in means and variances between the untransformed and transformed distributions. The mean $\left(\mu_{\varepsilon_t}\right)$ and variance $\left(\sigma_{\varepsilon_t}{}^2\right)$ of the untransformed Weibull distribution are, respectively, given in (14) and (15).Considering the unit mean assumption required for modeling with this class of distribution, we would calculate the theoretical values of $\mu_{\varepsilon_t}, \sigma_{\varepsilon_t}{}^2\ \mu_{y_t}$ and $\sigma_{y_t}{}^2$ using values of σ = 7, 8 , . . . , 99, 100 and corresponding values of n for which $\mu_{\varepsilon_t} = 1.0$ (Ohakwe and Ajibade, 2019). From (14), for $\mu_{\varepsilon_t} = 1.0$, without loss of generality, we would adopt $\mathbf{n} = \dfrac{\mathbf{1}}{\mathbf{\Gamma}\left(\mathbf{1}+\frac{1}{\boldsymbol{\sigma}}\right)}$ in order to maintain the values of the shape parameters as positive integers (Ohakwe and Ajibade, 2019). That is for every value of σ we would use the corresponding values of $n = \dfrac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$ for all the computations involving the untransformed and transformed distributions. Table 5 contains the means of the transformed and the untransformed distributions while their variances are contained in Table 6.

Considering that the interest in this study is to examine the effect of the various power transformations on the Weibull distributed error term as regards the mean and the variance, we will adopt the same measures used in Ohakwe and Ajibade (2019) which are the relative change in means and variances between the untransformed and the transformed distributions in measuring the effect of a transformation. The measures are given as follows; for the effect on the mean, the two variables of interest are μ_{ε_t} and μ_{y_t} and the relative change in mean (RCIM) is

$$RCIM = \frac{\mu_{y_t} - \mu_{\varepsilon_t}}{\mu_{\varepsilon_t}} = \mu_{y_t} - 1.0, \tag{26}$$

where RCIM > 0 indicates increase, RCIM = 0 indicates no change and RCIM < 0 indicates decrease in mean.

Furthermore, for the effect on the variance, the determinant variables are $\sigma_{\varepsilon_t}{}^2$ and $\sigma_{y_t}{}^2$ and the measure for the Relative Change in Variance (RCIV) between the transformed and the untransformed distributions is

$$\mathbf{RCIV} = \frac{\sigma_{y_t}{}^2 - \sigma_{\varepsilon_t}{}^2}{\sigma_{\varepsilon_t}{}^2}, \qquad (27)$$

where RCIV > 0 indicates increase, RCIV = 0 indicates no change and RCIV < 0 indicates decrease in variance. Whereas the theoretical means of the transformed distributions are approximately 1.0 to the nearest whole number for all $\sigma \geq 7$ as shown in Table 4, we therefore compute the RCIM and RCIV values for $\sigma \geq 7$. Furthermore, the plots of the variances of the untransformed and transformed distributions against the shape parameter values, $\sigma \geq 7$ are given in Figure 1 while the computations of the RCIV are contained in Table 6. Furthermore, plots of the RCIV against α are given in Figure 2. Finally, the mean values, minimum and maximum values of the RCIV for the various transformations are given in Table 7.

4 Results and Discussions

The results in Table 4 indicate that the unit-mean assumption is unaffected by the power transformations as investigated. This result is in agreement with the findings of Ohakwe *et al.* (2012) and Ohakwe and Ajibade, 2019). For the variances given in Table 5, the variances for the inverse (VIT), square (VST), inverse square (VIST), cube root (VCT) and inverse cube (VICT) transformed distributions are higher than the variance of the untransformed distribution (Vet) while those of the square root(VSRT), inverse square root(VISRT), cube root (VCRT) and inverse cube root (VICRT) are lower than that of the untransformed distribution. These higher and lower variances can be clearly seen in Figure 1 which is also supported by the results in Table 6.

In Figure 2, the RCIV for IT, ST, IST, CT and ICT are all greater than zero which indicate increased variances under such transformations while those for SRT, ISRT, CRT and ICRT are all less than zero which indicate reduced variances resulting from such transformations. However, based on the result on Tables 6 and 7 the highest magnitude of increases in variances are ICT, CT, IST, ST and IT in decreasing order with factors ranging from: 8.5800 – 96.7500, 7.1447 – 7.7630, 3.1860 – 10.9160, 2.7294 – 2.9456 and 0.03 – 0.6512 respectively. The RCIV values for SRT, ISRT, CRT and ICRT that indicated reduced variances are with factors: -0.8878 – (-0.8771), -0.8867 – (-0.8556), -0.7482 – (-0.7317) and -0.7445 – (-0.6579), respectively. However, it is noticeable that the magnitude of the decrease are approximately the same. Finally, it is important to mention that stability of the variances for all the transformations is achieved from the point, $\alpha \geq 17$, where the variances for all the transformations are all approximately zero.

5 Conclusion

The study discussed nine (9) power transformations of the two-parameter Weibull distribution of the multiplicative error model which have importance in presenting a new way of modeling random durations emanating from time varying events. The Weibull distribution is considered because of its flexibility to investigate the effects of transformations on the unit mean and variance of the error component of the multiplicative error model which requires a variance-stabilization. We obtained the pdfs, mean and variance of the transformed distribution. We observed that decreasing the values of scale parameter is meaningful and effective for the inverse and square root transformations Ozdemire (2017). The assumption of unit mean was verified for all transformations, whereas for the variance it was found that there exist relative increases for the inverse, square, inverse square, cube and inverse cube transformations. The paper finally concludes that the square root, inverse square root, cube root and inverse cube root would yield better results when using MEM with a Weibull distributed error component and where data transformation is deemed necessary to stabilize the variance of the data set.

References

Adejumo, A.O. and Ahmadu, A.O. (2016). A Study of the Slope of Cox Proportional Hazard and Weibull Models: Simulated and Real Life Data Approach, *Science World Journal*, 11 (3), 2016.

Akpanta, A.C. and Iwueze, I.S. (2009). On Applying the Bartlett Transformation Methods to Time Series Data, *Journal of Mathematical Sciences*, 20, (3), 171 – 199.

Andersen, T.G., Bollerslev, T., Diebold, F.X. and Ebens, H. (2001). The distribution of realized stock return volatility, *Journal of Financial Economics*, 61, 43-76.

Bandi, F.M. and Russell, J.R. (2006). Separating microstructure noise from volatility, *Journal of Financial Economics*, 79, 655 – 692.

Bartlett, M.S. (1947).The Use of Transformations, *Biometrics*, 3, 39 – 52.

Box, G.E.P. and Cox, D.R. (1964). An Analysis of Transformation, *Journal of Royal Statistical Society – Series B*, 26, 211 – 252.

Brownlees, C.T. Cipollini, F. and Gallo G.M. (2011). Multiplicative Error Models, Working paper, http:ssrn.com/abstract=1852285.

Chartfield, C. (2004). *The Analysis of Time Series: An Introduction*, 6th ed., Chapman and Hall, CRC Press, Boca Raton.

Dike, A.O., Otuonye, E.L. and Chikezie, D.C. (2016). The nth Power Transformation of the Error Component of the Multiplicative Time Series Model, *British Journal of Mathematics and Computer Science*, 18(1), 1 – 15.

Engle, R. and Russell, J.R. (1998). Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data, *Econometrica*, 66 (5), 1127 – 1162.

Engle, R. (2002). Dynamic Conditional Correlation: A Simple Class of Multivariate Generalized Autoregressive Conditional Heteroskedasticity Models, *Journal of Business & Economic Statistics*, 20, 339 – 350. DOI: https://doi.org/10.1198/073500102288618487.

Healy, M.J.R. and Taylor, L.R. (1962). Tables for Power-Law Transformations, *Biometrika*, 49, 557 – 559.

Ibe, G.C. and Nwosu, C.R. (2013).Study on the Error Component of Multiplicative Time Series Model Under Inverse Transformation, *American Journal of Mathematics and Statistics*, 3(6), 362 – 374.

Iwueze, I.S. (2007). Some Implications of Truncating the N(1, σ2) Distribution to the Left at Zero, *Journal of Applied Sciences*, 7(2), 189 - 195.

Manganelli, S. (2005). Duration, Volume and Volatility Impact of Trades, *Journal of Financial Markets*, 8, 377-399.

Moore, P.G. (1957). Transformations to Normality Using Fractional Powers of the Variables. *Journal the American Statistical Association*, 52, 237-246.

Nwosu, C.R., Iwueze, I.S. and Ohakwe, J. (2013). Condition for Successful Inverse Transformation of the Error Component of the Multiplicative Time Series Model, *Asian Journal of Applied Sciences*, 6(1), 1–15. DOI: 10.3923/ajaps.2013.1-15.

Ohakwe, J. (2013). The Effect of Inverse Transformation on the Unit Mean and Constant Variance Assumptions of a Multiplicative Error Model Whose Error Component has a Gamma Distribution, *Mathematical Theory and Modeling*, 3(3), 44 – 52.

Ohakwe, J. and Ajibade, F.B. (2019). The Impact of Power Transformations on the Parameters of the Gamma Distributed Error Component of a Multiplicative Error Model. *To appear in Benin Journal of Statistics, 2019.*

Ohakwe, J., Dike, O.A. and Akpanta, A.C. (2012). The Implication of Square Root Transformation on a Gamma Distributed Error Component of a Multiplicative Time Series Model, *Proceedings of African Regional Conference on Sustainable Development*, University of Calabar, Nigeria, 2012, 6 (4), 65 – 78.

Ohakwe, J, Iwueze, I.S. and Otuonye, E.L. (2013). The Exact Region for the Successful Application of Square Root Transformation in Time Series Decomposition using the Multiplicative Model, *International Journal of Basic Science and Technology*, 4(1), 1 – 11.

Ohakwe, J., Akpanta, A.C. and Dike, O.A. (2012). Unit Mean and Constant Variance of the Generalized Gamma Distribution after Square Root Transformation in Statistical Modeling, *Mathematical Theory and Modeling*, 2 (12), 22 - 34.

Osborne, J. (2002). Notes on the use of Data Transformations, *Journal of Practice Assessment, Research and Evaluation*, 8(6), 1 – 10.

Otuonye, E.L., Iwueze, I.S. and Ohakwe, J. (2011). The Effect of Square Root Transformation on the Error Component of the Multiplicative Time Series Model, *International Journal of Statistics and Systems*, 6(4), 461 – 476.

Ozdemir, O. (2017). Power Transformations for Families of Statistical Distributions to Satisfy Normality, *International Journal of Economics and Statistics*, 5, 1 – 4.

Patton, A. (2011). Volatility Forecast Comparison Using Imperfect Volatility Proxies, *Journal of Econometrics*, 160(1), 246 – 256.

Ramachandran, K.M. and Tsokos, C.P. (2009). *Mathematical Statistics with Applications*, Academic Press, London.

Russell, J.R. and Engle, R.F. (2010). Analysis of *high-frequency data*, In: Ait-Sahalia, Y. and L.P. Hansen, (eds), *Handbook of Financial Econometrics, Vol. 1, Tools and Techniques*, 383 – 427, North Holland Publishing, Amsterdam.

Stanley, C.R. (2006). Numerical Transformation of Geochemical Data, *Geochemistry: Exploration, Environment, Analysis,* 6, 69-78.

Tukey, J.W. (1957). On the Comparative Anatomy of Transformations, *Annals of Mathematical Statistics*, 28, 602-632.

Table 4 : Means of the untransformed and Transformed Distributions

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	μ_{ε_t}	Mean of the transformed Distribution $\left(\mu_{y_t}\right)$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
7	1.0690	1	1.0344	0.9962	1.0123	1.0282	1.1165	0.9966	1.0072	1.0825	1.2758
8	1.0619	1	1.0262	0.9971	1.0094	1.0220	1.0868	0.9974	1.0055	1.0643	1.1981
9	1.0560	1	1.0206	0.9977	1.0075	1.0177	1.0673	0.9979	1.0044	1.0516	1.1499
10	1.0511	1	1.0166	0.9981	1.0061	1.0145	1.0537	0.9983	1.0036	1.0423	1.1177
11	1.0470	1	1.0137	0.9984	1.0050	1.0121	1.0439	0.9986	1.0029	1.0354	1.0950
12	1.0435	1	1.0115	0.9987	1.0042	1.0102	1.0366	0.9988	1.0025	1.0300	1.0784
13	1.0405	1	1.0098	0.9988	1.0036	1.0088	1.0310	0.9990	1.0021	1.0258	1.0658
14	1.0379	1	1.0084	0.9990	1.0031	1.0076	1.0265	0.9991	1.0018	1.0224	1.0561
15	1.0356	1	1.0073	0.9991	1.0027	1.0067	1.0230	0.9992	1.0016	1.0196	1.0484
16	1.0335	1	1.0065	0.9992	1.0024	1.0059	1.0201	0.9993	1.0014	1.0174	1.0422
17	1.0317	1	1.0057	0.9993	1.0021	1.0053	1.0178	0.9994	1.0012	1.0155	1.0371
18	1.0300	1	1.0051	0.9994	1.0019	1.0047	1.0158	0.9995	1.0011	1.0139	1.0329
19	1.0286	1	1.0046	0.9995	1.0017	1.0042	1.0142	0.9995	1.0010	1.0125	1.0294
20	1.0272	1	1.0041	0.9995	1.0015	1.0038	1.0128	0.9996	1.0009	1.0113	1.0264
21	1.0260	1	1.0037	0.9995	1.0014	1.0035	1.0115	0.9996	1.0008	1.0103	1.0238
22	1.0249	1	1.0034	0.9996	1.0013	1.0032	1.0105	0.9996	1.0007	1.0094	1.0216
23	1.0239	1	1.0031	0.9996	1.0012	1.0029	1.0096	0.9997	1.0007	1.0086	1.0197
24	1.0229	1	1.0029	0.9997	1.0011	1.0027	1.0088	0.9997	1.0006	1.0080	1.0181
25	1.0220	1	1.0026	0.9997	1.0010	1.0025	1.0081	0.9997	1.0006	1.0074	1.0166
26	1.0212	1	1.0024	0.9997	1.0009	1.0023	1.0075	0.9997	1.0005	1.0068	1.0153
27	1.0205	1	1.0023	0.9997	1.0008	1.0021	1.0069	0.9998	1.0005	1.0063	1.0142
28	1.0198	1	1.0021	0.9997	1.0008	1.0020	1.0064	0.9998	1.0005	1.0059	1.0132
29	1.0191	1	1.0020	0.9998	1.0007	1.0019	1.0060	0.9998	1.0004	1.0055	1.0122
30	1.0185	1	1.0018	0.9998	1.0007	1.0017	1.0056	0.9998	1.0004	1.0052	1.0114
31	1.0179	1	1.0017	0.9998	1.0006	1.0016	1.0052	0.9998	1.0004	1.0048	1.0107
32	1.0174	1	1.0016	0.9998	1.0006	1.0015	1.0049	0.9998	1.0004	1.0046	1.0100
33	1.0169	1	1.0015	0.9998	1.0006	1.0014	1.0046	0.9998	1.0003	1.0043	1.0094

Table 4 Continues

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	μ_{ε_t}	Mean of the transformed Distribution (μ_{y_t})								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
34	1.0164	1	1.0014	0.9998	1.0005	1.0014	1.0043	0.9998	1.0003	1.0040	1.0088
35	1.0137	1	1.0010	0.9999	1.0004	1.0009	1.0030	0.9999	1.0002	1.0028	1.0060
36	1.0134	1	1.0009	0.9999	1.0003	1.0009	1.0028	0.9999	1.0002	1.0027	1.0057
37	1.0131	1	1.0009	0.9999	1.0003	1.0009	1.0027	0.9999	1.0002	1.0026	1.0055
38	1.0128	1	1.0009	0.9999	1.0003	1.0008	1.0026	0.9999	1.0002	1.0024	1.0052
39	1.0125	1	1.0008	0.9999	1.0003	1.0008	1.0025	0.9999	1.0002	1.0023	1.0050
40	1.0122	1	1.0008	0.9999	1.0003	1.0008	1.0024	0.9999	1.0002	1.0022	1.0048
41	1.0120	1	1.0007	0.9999	1.0003	1.0007	1.0023	0.9999	1.0002	1.0021	1.0046
42	1.0117	1	1.0007	0.9999	1.0003	1.0007	1.0022	0.9999	1.0002	1.0021	1.0044
43	1.0115	1	1.0007	0.9999	1.0003	1.0007	1.0021	0.9999	1.0002	1.0020	1.0042
44	1.0113	1	1.0007	0.9999	1.0002	1.0006	1.0020	0.9999	1.0001	1.0019	1.0040
45	1.0111	1	1.0006	0.9999	1.0002	1.0006	1.0019	0.9999	1.0001	1.0018	1.0039
46	1.0109	1	1.0006	0.9999	1.0002	1.0006	1.0018	0.9999	1.0001	1.0018	1.0037
47	1.0107	1	1.0006	0.9999	1.0002	1.0006	1.0018	0.9999	1.0001	1.0017	1.0036
48	1.0105	1	1.0006	0.9999	1.0002	1.0005	1.0017	0.9999	1.0001	1.0016	1.0035
49	1.0103	1	1.0005	0.9999	1.0002	1.0005	1.0016	0.9999	1.0001	1.0016	1.0033
50	1.0101	1	1.0005	0.9999	1.0002	1.0005	1.0016	0.9999	1.0001	1.0015	1.0032
51	1.0099	1	1.0005	0.9999	1.0002	1.0005	1.0015	0.9999	1.0001	1.0015	1.0031
52	1.0098	1	1.0005	0.9999	1.0002	1.0005	1.0015	0.9999	1.0001	1.0014	1.0030
53	1.0096	1	1.0005	0.9999	1.0002	1.0005	1.0014	0.9999	1.0001	1.0014	1.0029
54	1.0094	1	1.0005	0.9999	1.0002	1.0004	1.0014	0.9999	1.0001	1.0013	1.0028
55	1.0093	1	1.0004	0.9999	1.0002	1.0004	1.0013	1.0000	1.0001	1.0013	1.0027
56	1.0091	1	1.0004	0.9999	1.0002	1.0004	1.0013	1.0000	1.0001	1.0012	1.0026
57	1.0090	1	1.0004	0.9999	1.0002	1.0004	1.0013	1.0000	1.0001	1.0012	1.0025
58	1.0089	1	1.0004	1.0000	1.0002	1.0004	1.0012	1.0000	1.0001	1.0012	1.0025
59	1.0087	1	1.0004	1.0000	1.0001	1.0004	1.0012	1.0000	1.0001	1.0011	1.0024
60	1.0086	1	1.0004	1.0000	1.0001	1.0004	1.0011	1.0000	1.0001	1.0011	1.0023

Table 4 Continues

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	μ_{ε_t}	Mean of the transformed Distribution $\left(\mu_{y_t}\right)$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
61	1.0085	1	1.0004	1.0000	1.0001	1.0004	1.0011	1.0000	1.0001	1.0011	1.0022
62	1.0083	1	1.0004	1.0000	1.0001	1.0003	1.0011	1.0000	1.0001	1.0010	1.0022
63	1.0082	1	1.0003	1.0000	1.0001	1.0003	1.0010	1.0000	1.0001	1.0010	1.0021
64	1.0089	1	1.0004	1.0000	1.0002	1.0004	1.0012	1.0000	1.0001	1.0012	1.0025
65	1.0087	1	1.0004	1.0000	1.0001	1.0004	1.0012	1.0000	1.0001	1.0011	1.0024
66	1.0086	1	1.0004	1.0000	1.0001	1.0004	1.0011	1.0000	1.0001	1.0011	1.0023
67	1.0085	1	1.0004	1.0000	1.0001	1.0004	1.0011	1.0000	1.0001	1.0011	1.0022
68	1.0083	1	1.0004	1.0000	1.0001	1.0003	1.0011	1.0000	1.0001	1.0010	1.0022
69	1.0082	1	1.0003	1.0000	1.0001	1.0003	1.0010	1.0000	1.0001	1.0010	1.0021
70	1.0081	1	1.0003	1.0000	1.0001	1.0003	1.0010	1.0000	1.0001	1.0010	1.0020
71	1.0080	1	1.0003	1.0000	1.0001	1.0003	1.0010	1.0000	1.0001	1.0010	1.0020
72	1.0079	1	1.0003	1.0000	1.0001	1.0003	1.0010	1.0000	1.0001	1.0009	1.0019
73	1.0078	1	1.0003	1.0000	1.0001	1.0003	1.0009	1.0000	1.0001	1.0009	1.0019
74	1.0077	1	1.0003	1.0000	1.0001	1.0003	1.0009	1.0000	1.0001	1.0009	1.0018
75	1.0076	1	1.0003	1.0000	1.0001	1.0003	1.0009	1.0000	1.0001	1.0009	1.0018
76	1.0075	1	1.0003	1.0000	1.0001	1.0003	1.0009	1.0000	1.0001	1.0008	1.0017
77	1.0074	1	1.0003	1.0000	1.0001	1.0003	1.0008	1.0000	1.0001	1.0008	1.0017
78	1.0073	1	1.0003	1.0000	1.0001	1.0003	1.0008	1.0000	1.0001	1.0008	1.0016
79	1.0072	1	1.0003	1.0000	1.0001	1.0003	1.0008	1.0000	1.0001	1.0008	1.0016
80	1.0071	1	1.0003	1.0000	1.0001	1.0003	1.0008	1.0000	1.0001	1.0008	1.0016
81	1.0070	1	1.0003	1.0000	1.0001	1.0002	1.0008	1.0000	1.0001	1.0007	1.0015
82	1.0069	1	1.0002	1.0000	1.0001	1.0002	1.0007	1.0000	1.0001	1.0007	1.0015
83	1.0069	1	1.0002	1.0000	1.0001	1.0002	1.0007	1.0000	1.0001	1.0007	1.0015
84	1.0068	1	1.0002	1.0000	1.0001	1.0002	1.0007	1.0000	1.0001	1.0007	1.0014
85	1.0067	1	1.0002	1.0000	1.0001	1.0002	1.0007	1.0000	1.0001	1.0007	1.0014
86	1.0066	1	1.0002	1.0000	1.0001	1.0002	1.0007	1.0000	1.0000	1.0007	1.0014
87	1.0065	1	1.0002	1.0000	1.0001	1.0002	1.0007	1.0000	1.0000	1.0006	1.0013

Table 4 Continues											
σ	$\mathbf{n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}}$	$\boldsymbol{\mu_{\varepsilon_t}}$	**Mean of the transformed Distribution** $\left(\boldsymbol{\mu_{y_t}}\right)$								
			IT	**SRT**	**ISRT**	**ST**	**IST**	**CRT**	**ICRT**	**CT**	**ICT**
88	1.0065	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0006	1.0013
89	1.0064	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0006	1.0013
90	1.0063	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0006	1.0012
91	1.0063	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0006	1.0012
92	1.0062	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0006	1.0012
93	1.0061	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0006	1.0012
94	1.0061	1	1.0002	1.0000	1.0001	1.0002	1.0006	1.0000	1.0000	1.0005	1.0011
95	1.0060	1	1.0002	1.0000	1.0001	1.0002	1.0005	1.0000	1.0000	1.0005	1.0011
96	1.0059	1	1.0002	1.0000	1.0001	1.0002	1.0005	1.0000	1.0000	1.0005	1.0011
97	1.0059	1	1.0002	1.0000	1.0001	1.0002	1.0005	1.0000	1.0000	1.0005	1.0011
98	1.0058	1	1.0002	1.0000	1.0001	1.0002	1.0005	1.0000	1.0000	1.0005	1.0010
99	1.0058	1	1.0002	1.0000	1.0001	1.0002	1.0005	1.0000	1.0000	1.0005	1.0010
100	1.0057	1	1.0002	1.0000	1.0001	1.0002	1.0005	1.0000	1.0000	1.0005	1.0010

Table 5 : Variance of the untransformed and Transformed Distributions

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	Variance of the transformed Distribution $\left(\sigma^2_{y_t}\right)$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
7	1.0690	0.0282	0.0466	0.0076	0.0097	0.1059	0.3364	0.0035	0.0041	0.2428	2.7597
8	1.0619	0.0220	0.0338	0.0059	0.0072	0.0822	0.2130	0.0027	0.0031	0.1848	1.0936
9	1.0560	0.0177	0.0256	0.0047	0.0056	0.0658	0.1482	0.0021	0.0024	0.1461	0.6096
10	1.0511	0.0145	0.0201	0.0038	0.0045	0.0540	0.1096	0.0017	0.0019	0.1188	0.3953
11	1.0470	0.0121	0.0163	0.0032	0.0037	0.0451	0.0846	0.0014	0.0016	0.0987	0.2798
12	1.0435	0.0102	0.0134	0.0027	0.0031	0.0383	0.0674	0.0012	0.0013	0.0835	0.2097
13	1.0405	0.0088	0.0113	0.0023	0.0026	0.0330	0.0551	0.0010	0.0011	0.0716	0.1637
14	1.0379	0.0076	0.0096	0.0020	0.0022	0.0287	0.0459	0.0009	0.0010	0.0622	0.1317
15	1.0356	0.0067	0.0083	0.0017	0.0019	0.0252	0.0389	0.0008	0.0008	0.0545	0.1085
16	1.0335	0.0059	0.0072	0.0015	0.0017	0.0223	0.0334	0.0007	0.0007	0.0483	0.0910
17	1.0317	0.0053	0.0063	0.0014	0.0015	0.0199	0.0290	0.0006	0.0007	0.0430	0.0776
18	1.0300	0.0047	0.0056	0.0012	0.0013	0.0178	0.0254	0.0005	0.0006	0.0386	0.0669
19	1.0286	0.0042	0.0050	0.0011	0.0012	0.0161	0.0225	0.0005	0.0005	0.0348	0.0584
20	1.0272	0.0038	0.0045	0.0010	0.0011	0.0146	0.0200	0.0004	0.0005	0.0316	0.0514
21	1.0260	0.0035	0.0040	0.0009	0.0010	0.0133	0.0179	0.0004	0.0004	0.0288	0.0457
22	1.0249	0.0032	0.0037	0.0008	0.0009	0.0122	0.0162	0.0004	0.0004	0.0264	0.0408
23	1.0239	0.0029	0.0033	0.0008	0.0008	0.0112	0.0147	0.0003	0.0004	0.0242	0.0367
24	1.0229	0.0027	0.0031	0.0007	0.0007	0.0103	0.0133	0.0003	0.0003	0.0224	0.0332
25	1.0220	0.0025	0.0028	0.0006	0.0007	0.0095	0.0122	0.0003	0.0003	0.0207	0.0302
26	1.0212	0.0023	0.0026	0.0006	0.0006	0.0088	0.0112	0.0003	0.0003	0.0192	0.0276
27	1.0205	0.0021	0.0024	0.0005	0.0006	0.0082	0.0103	0.0002	0.0003	0.0179	0.0253
28	1.0198	0.0020	0.0022	0.0005	0.0005	0.0077	0.0095	0.0002	0.0002	0.0167	0.0233
29	1.0191	0.0019	0.0021	0.0005	0.0005	0.0072	0.0089	0.0002	0.0002	0.0156	0.0215
30	1.0185	0.0017	0.0019	0.0004	0.0005	0.0067	0.0082	0.0002	0.0002	0.0146	0.0199
31	1.0179	0.0016	0.0018	0.0004	0.0004	0.0063	0.0077	0.0002	0.0002	0.0137	0.0185
32	1.0174	0.0015	0.0017	0.0004	0.0004	0.0059	0.0072	0.0002	0.0002	0.0129	0.0173
33	1.0169	0.0014	0.0016	0.0004	0.0004	0.0056	0.0067	0.0002	0.0002	0.0122	0.0161

Table 5 Continues

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	Variance of the transformed Distribution $\left(\sigma^2_{y_t}\right)$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
34	1.0164	0.0014	0.0015	0.0003	0.0004	0.0053	0.0063	0.0002	0.0002	0.0115	0.0151
35	1.0137	0.0013	0.0014	0.0003	0.0003	0.0050	0.0059	0.0001	0.0002	0.0109	0.0142
36	1.0134	0.0012	0.0013	0.0003	0.0003	0.0047	0.0056	0.0001	0.0001	0.0103	0.0133
37	1.0131	0.0012	0.0013	0.0003	0.0003	0.0045	0.0053	0.0001	0.0001	0.0098	0.0125
38	1.0128	0.0011	0.0012	0.0003	0.0003	0.0043	0.0050	0.0001	0.0001	0.0093	0.0118
39	1.0125	0.0010	0.0011	0.0003	0.0003	0.0040	0.0047	0.0001	0.0001	0.0088	0.0112
40	1.0122	0.0010	0.0011	0.0003	0.0003	0.0038	0.0045	0.0001	0.0001	0.0084	0.0106
41	1.0120	0.0009	0.0010	0.0002	0.0002	0.0037	0.0043	0.0001	0.0001	0.0080	0.0100
42	1.0117	0.0009	0.0010	0.0002	0.0002	0.0035	0.0040	0.0001	0.0001	0.0077	0.0095
43	1.0115	0.0009	0.0009	0.0002	0.0002	0.0033	0.0038	0.0001	0.0001	0.0073	0.0091
44	1.0113	0.0008	0.0009	0.0002	0.0002	0.0032	0.0037	0.0001	0.0001	0.0070	0.0086
45	1.0111	0.0008	0.0008	0.0002	0.0002	0.0031	0.0035	0.0001	0.0001	0.0067	0.0082
46	1.0109	0.0008	0.0008	0.0002	0.0002	0.0029	0.0033	0.0001	0.0001	0.0064	0.0078
47	1.0107	0.0007	0.0008	0.0002	0.0002	0.0028	0.0032	0.0001	0.0001	0.0062	0.0075
48	1.0105	0.0007	0.0007	0.0002	0.0002	0.0027	0.0031	0.0001	0.0001	0.0059	0.0072
49	1.0103	0.0007	0.0007	0.0002	0.0002	0.0026	0.0029	0.0001	0.0001	0.0057	0.0069
50	1.0101	0.0006	0.0007	0.0002	0.0002	0.0025	0.0028	0.0001	0.0001	0.0055	0.0066
51	1.0099	0.0006	0.0007	0.0002	0.0002	0.0024	0.0027	0.0001	0.0001	0.0053	0.0063
52	1.0098	0.0006	0.0006	0.0001	0.0002	0.0023	0.0026	0.0001	0.0001	0.0051	0.0060
53	1.0096	0.0006	0.0006	0.0001	0.0001	0.0022	0.0025	0.0001	0.0001	0.0049	0.0058
54	1.0094	0.0005	0.0006	0.0001	0.0001	0.0021	0.0024	0.0001	0.0001	0.0047	0.0056
55	1.0093	0.0005	0.0006	0.0001	0.0001	0.0021	0.0023	0.0001	0.0001	0.0046	0.0054
56	1.0091	0.0005	0.0005	0.0001	0.0001	0.0020	0.0022	0.0001	0.0001	0.0044	0.0052
57	1.0090	0.0005	0.0005	0.0001	0.0001	0.0019	0.0021	0.0001	0.0001	0.0043	0.0050
58	1.0089	0.0005	0.0005	0.0001	0.0001	0.0019	0.0021	0.0001	0.0001	0.0041	0.0048
59	1.0087	0.0005	0.0005	0.0001	0.0001	0.0018	0.0020	0.0001	0.0001	0.0040	0.0046
60	1.0086	0.0004	0.0005	0.0001	0.0001	0.0017	0.0019	0.0001	0.0001	0.0039	0.0045

Table 5 Continues											
σ	$n = \frac{1}{\Gamma(1+\frac{1}{\sigma})}$	$\sigma^2_{\varepsilon_t}$	**Variance of the transformed Distribution** $(\sigma^2_{y_t})$								
			IT	**SRT**	**ISRT**	**ST**	**IST**	**CRT**	**ICRT**	**CT**	**ICT**
61	1.0085	0.0004	0.0005	0.0001	0.0001	0.0017	0.0019	0.0000	0.0000	0.0037	0.0043
62	1.0083	0.0004	0.0004	0.0001	0.0001	0.0016	0.0018	0.0000	0.0000	0.0036	0.0042
63	1.0082	0.0004	0.0004	0.0001	0.0001	0.0016	0.0017	0.0000	0.0000	0.0035	0.0040
64	1.0089	0.0004	0.0004	0.0001	0.0001	0.0015	0.0017	0.0000	0.0000	0.0034	0.0039
65	1.0087	0.0004	0.0004	0.0001	0.0001	0.0015	0.0016	0.0000	0.0000	0.0033	0.0038
66	1.0086	0.0004	0.0004	0.0001	0.0001	0.0014	0.0016	0.0000	0.0000	0.0032	0.0037
67	1.0085	0.0004	0.0004	0.0001	0.0001	0.0014	0.0015	0.0000	0.0000	0.0031	0.0036
68	1.0083	0.0003	0.0004	0.0001	0.0001	0.0014	0.0015	0.0000	0.0000	0.0030	0.0034
69	1.0082	0.0003	0.0004	0.0001	0.0001	0.0013	0.0014	0.0000	0.0000	0.0029	0.0033
70	1.0081	0.0003	0.0003	0.0001	0.0001	0.0013	0.0014	0.0000	0.0000	0.0029	0.0032
71	1.0080	0.0003	0.0003	0.0001	0.0001	0.0013	0.0014	0.0000	0.0000	0.0028	0.0031
72	1.0079	0.0003	0.0003	0.0001	0.0001	0.0012	0.0013	0.0000	0.0000	0.0027	0.0031
73	1.0078	0.0003	0.0003	0.0001	0.0001	0.0012	0.0013	0.0000	0.0000	0.0026	0.0030
74	1.0077	0.0003	0.0003	0.0001	0.0001	0.0012	0.0013	0.0000	0.0000	0.0026	0.0029
75	1.0076	0.0003	0.0003	0.0001	0.0001	0.0011	0.0012	0.0000	0.0000	0.0025	0.0028
76	1.0075	0.0003	0.0003	0.0001	0.0001	0.0011	0.0012	0.0000	0.0000	0.0024	0.0027
77	1.0074	0.0003	0.0003	0.0001	0.0001	0.0011	0.0012	0.0000	0.0000	0.0024	0.0027
78	1.0073	0.0003	0.0003	0.0001	0.0001	0.0010	0.0011	0.0000	0.0000	0.0023	0.0026
79	1.0072	0.0003	0.0003	0.0001	0.0001	0.0010	0.0011	0.0000	0.0000	0.0023	0.0025
80	1.0071	0.0003	0.0003	0.0001	0.0001	0.0010	0.0011	0.0000	0.0000	0.0022	0.0025
81	1.0070	0.0002	0.0003	0.0001	0.0001	0.0010	0.0010	0.0000	0.0000	0.0021	0.0024
82	1.0069	0.0002	0.0002	0.0001	0.0001	0.0009	0.0010	0.0000	0.0000	0.0021	0.0023
83	1.0069	0.0002	0.0002	0.0001	0.0001	0.0009	0.0010	0.0000	0.0000	0.0020	0.0023
84	1.0068	0.0002	0.0002	0.0001	0.0001	0.0009	0.0010	0.0000	0.0000	0.0020	0.0022
85	1.0067	0.0002	0.0002	0.0001	0.0001	0.0009	0.0009	0.0000	0.0000	0.0020	0.0022
86	1.0066	0.0002	0.0002	0.0001	0.0001	0.0009	0.0009	0.0000	0.0000	0.0019	0.0021
87	1.0065	0.0002	0.0002	0.0001	0.0001	0.0008	0.0009	0.0000	0.0000	0.0019	0.0021

Table 5 Continues

σ	$n = \frac{1}{\Gamma(1+\frac{1}{\sigma})}$	$\sigma^2_{\varepsilon_t}$	Variance of the transformed Distribution $(\sigma^2_{y_t})$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
88	1.0065	0.0002	0.0002	0.0001	0.0001	0.0008	0.0009	0.0000	0.0000	0.0018	0.0020
89	1.0064	0.0002	0.0002	0.0001	0.0001	0.0008	0.0009	0.0000	0.0000	0.0018	0.0020
90	1.0063	0.0002	0.0002	0.0001	0.0001	0.0008	0.0008	0.0000	0.0000	0.0017	0.0019
91	1.0063	0.0002	0.0002	0.0000	0.0001	0.0008	0.0008	0.0000	0.0000	0.0017	0.0019
92	1.0062	0.0002	0.0002	0.0000	0.0000	0.0008	0.0008	0.0000	0.0000	0.0017	0.0018
93	1.0061	0.0002	0.0002	0.0000	0.0000	0.0007	0.0008	0.0000	0.0000	0.0016	0.0018
94	1.0061	0.0002	0.0002	0.0000	0.0000	0.0007	0.0008	0.0000	0.0000	0.0016	0.0018
95	1.0060	0.0002	0.0002	0.0000	0.0000	0.0007	0.0008	0.0000	0.0000	0.0016	0.0017
96	1.0059	0.0002	0.0002	0.0000	0.0000	0.0007	0.0007	0.0000	0.0000	0.0015	0.0017
97	1.0059	0.0002	0.0002	0.0000	0.0000	0.0007	0.0007	0.0000	0.0000	0.0015	0.0017
98	1.0058	0.0002	0.0002	0.0000	0.0000	0.0007	0.0007	0.0000	0.0000	0.0015	0.0016
99	1.0058	0.0002	0.0002	0.0000	0.0000	0.0007	0.0007	0.0000	0.0000	0.0014	0.0016
100	1.0057	0.0002	0.0002	0.0000	0.0000	0.0006	0.0007	0.0000	0.0000	0.0014	0.0016

Table 6 : Relative Change in Variance (RCIV) of the untransformed and Transformed Distributions

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	$RCIV = \frac{\sigma_{y_t}^2 - \sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
7	1.0690	0.0282	0.6512	-0.7317	-0.6579	2.7504	10.9163	-0.8771	-0.8556	7.5997	96.7543
8	1.0619	0.0220	0.5340	-0.7332	-0.6713	2.7345	8.6762	-0.8782	-0.8601	7.3938	48.6808
9	1.0560	0.0177	0.4522	-0.7345	-0.6812	2.7294	7.3936	-0.8791	-0.8635	7.2747	33.5309
10	1.0511	0.0145	0.3920	-0.7356	-0.6889	2.7303	6.5699	-0.8799	-0.8662	7.2061	26.3111
11	1.0470	0.0121	0.3459	-0.7367	-0.6951	2.7346	5.9991	-0.8806	-0.8683	7.1685	22.1508
12	1.0435	0.0102	0.3095	-0.7375	-0.7001	2.7406	5.5817	-0.8811	-0.8701	7.1503	19.4705
13	1.0405	0.0088	0.2799	-0.7383	-0.7043	2.7476	5.2639	-0.8817	-0.8716	7.1447	17.6113
14	1.0379	0.0076	0.2555	-0.7390	-0.7078	2.7550	5.0144	-0.8821	-0.8729	7.1474	16.2516
15	1.0356	0.0067	0.2350	-0.7396	-0.7108	2.7625	4.8134	-0.8825	-0.8740	7.1555	15.2172
16	1.0335	0.0059	0.2176	-0.7402	-0.7135	2.7699	4.6482	-0.8828	-0.8750	7.1671	14.4057
17	1.0317	0.0053	0.2025	-0.7407	-0.7157	2.7772	4.5102	-0.8832	-0.8758	7.1811	13.7531
18	1.0300	0.0047	0.1894	-0.7411	-0.7178	2.7842	4.3931	-0.8834	-0.8766	7.1964	13.2176
19	1.0286	0.0042	0.1779	-0.7415	-0.7196	2.7909	4.2927	-0.8837	-0.8772	7.2126	12.7708
20	1.0272	0.0038	0.1677	-0.7419	-0.7212	2.7974	4.2056	-0.8839	-0.8778	7.2292	12.3926
21	1.0260	0.0035	0.1586	-0.7422	-0.7226	2.8035	4.1294	-0.8842	-0.8784	7.2460	12.0687
22	1.0249	0.0032	0.1505	-0.7425	-0.7239	2.8094	4.0621	-0.8844	-0.8788	7.2626	11.7881
23	1.0239	0.0029	0.1431	-0.7428	-0.7251	2.8149	4.0023	-0.8845	-0.8793	7.2790	11.5430
24	1.0229	0.0027	0.1364	-0.7431	-0.7262	2.8202	3.9489	-0.8847	-0.8797	7.2951	11.3270
25	1.0220	0.0025	0.1304	-0.7433	-0.7272	2.8253	3.9007	-0.8849	-0.8801	7.3108	11.1354
26	1.0212	0.0023	0.1248	-0.7436	-0.7281	2.8301	3.8572	-0.8850	-0.8804	7.3261	10.9643
27	1.0205	0.0021	0.1197	-0.7438	-0.7289	2.8346	3.8177	-0.8851	-0.8807	7.3409	10.8105
28	1.0198	0.0020	0.1150	-0.7440	-0.7297	2.8389	3.7816	-0.8853	-0.8810	7.3553	10.6717
29	1.0191	0.0019	0.1106	-0.7442	-0.7304	2.8431	3.7486	-0.8854	-0.8813	7.3692	10.5457
30	1.0185	0.0017	0.1066	-0.7444	-0.7311	2.8470	3.7182	-0.8855	-0.8816	7.3826	10.4309
31	1.0179	0.0016	0.1028	-0.7445	-0.7317	2.8508	3.6902	-0.8856	-0.8818	7.3956	10.3258
32	1.0174	0.0015	0.0994	-0.7447	-0.7323	2.8544	3.6643	-0.8857	-0.8820	7.4081	10.2293

Table 6 Continues											
σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	$RCIV = \frac{\sigma_{y_t}^2 - \sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$								
			IT	**SRT**	**ISRT**	**ST**	**IST**	**CRT**	**ICRT**	**CT**	**ICT**
33	1.0169	0.0014	0.0961	-0.7448	-0.7329	2.8578	3.6402	-0.8858	-0.8822	7.4202	10.1405
34	1.0164	0.0014	0.0930	-0.7450	-0.7334	2.8611	3.6178	-0.8859	-0.8824	7.4319	10.0583
35	1.0137	0.0013	0.0902	-0.7451	-0.7339	2.8642	3.5969	-0.8859	-0.8826	7.4431	9.9821
36	1.0134	0.0012	0.0875	-0.7452	-0.7344	2.8672	3.5774	-0.8860	-0.8828	7.4540	9.9114
37	1.0131	0.0012	0.0849	-0.7454	-0.7348	2.8701	3.5591	-0.8861	-0.8830	7.4645	9.8454
38	1.0128	0.0011	0.0825	-0.7455	-0.7352	2.8728	3.5419	-0.8862	-0.8831	7.4747	9.7838
39	1.0125	0.0010	0.0803	-0.7456	-0.7356	2.8755	3.5258	-0.8862	-0.8833	7.4844	9.7261
40	1.0122	0.0010	0.0781	-0.7457	-0.7360	2.8780	3.5105	-0.8863	-0.8834	7.4939	9.6720
41	1.0120	0.0009	0.0761	-0.7458	-0.7363	2.8805	3.4962	-0.8864	-0.8836	7.5031	9.6212
42	1.0117	0.0009	0.0742	-0.7459	-0.7366	2.8828	3.4826	-0.8864	-0.8837	7.5119	9.5733
43	1.0115	0.0009	0.0723	-0.7460	-0.7370	2.8851	3.4697	-0.8865	-0.8838	7.5205	9.5282
44	1.0113	0.0008	0.0706	-0.7461	-0.7373	2.8873	3.4575	-0.8865	-0.8839	7.5287	9.4855
45	1.0111	0.0008	0.0689	-0.7461	-0.7376	2.8894	3.4459	-0.8866	-0.8840	7.5367	9.4452
46	1.0109	0.0008	0.0673	-0.7462	-0.7378	2.8914	3.4349	-0.8866	-0.8841	7.5445	9.4069
47	1.0107	0.0007	0.0658	-0.7463	-0.7381	2.8933	3.4244	-0.8867	-0.8842	7.5520	9.3707
48	1.0105	0.0007	0.0644	-0.7464	-0.7383	2.8952	3.4144	-0.8867	-0.8843	7.5593	9.3362
49	1.0103	0.0007	0.0630	-0.7464	-0.7386	2.8971	3.4049	-0.8868	-0.8844	7.5664	9.3034
50	1.0101	0.0006	0.0616	-0.7465	-0.7388	2.8988	3.3958	-0.8868	-0.8845	7.5732	9.2722
51	1.0099	0.0006	0.0604	-0.7466	-0.7390	2.9005	3.3871	-0.8868	-0.8846	7.5798	9.2424
52	1.0098	0.0006	0.0592	-0.7466	-0.7393	2.9022	3.3787	-0.8869	-0.8847	7.5863	9.2140
53	1.0096	0.0006	0.0580	-0.7467	-0.7395	2.9037	3.3708	-0.8869	-0.8848	7.5925	9.1868
54	1.0094	0.0005	0.0568	-0.7468	-0.7397	2.9053	3.3631	-0.8869	-0.8848	7.5986	9.1608
55	1.0093	0.0005	0.0558	-0.7468	-0.7399	2.9068	3.3558	-0.8870	-0.8849	7.6045	9.1360
56	1.0091	0.0005	0.0547	-0.7469	-0.7400	2.9082	3.3487	-0.8870	-0.8850	7.6103	9.1121
57	1.0090	0.0005	0.0537	-0.7469	-0.7402	2.9096	3.3419	-0.8870	-0.8851	7.6158	9.0893
58	1.0089	0.0005	0.0527	-0.7470	-0.7404	2.9110	3.3354	-0.8871	-0.8851	7.6213	9.0673

Table 6 Continues

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	$RCIV = \frac{\sigma_{y_t}^2 - \sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$								
			IT	SRT	ISRT	ST	IST	CRT	ICRT	CT	ICT
59	1.0087	0.0005	0.0518	-0.7470	-0.7406	2.9123	3.3291	-0.8871	-0.8852	7.6265	9.0462
60	1.0086	0.0004	0.0509	-0.7471	-0.7407	2.9136	3.3231	-0.8871	-0.8853	7.6317	9.0260
61	1.0085	0.0004	0.0500	-0.7471	-0.7409	2.9149	3.3172	-0.8872	-0.8853	7.6367	9.0065
62	1.0083	0.0004	0.0492	-0.7472	-0.7410	2.9161	3.3116	-0.8872	-0.8854	7.6415	8.9877
63	1.0082	0.0004	0.0484	-0.7472	-0.7412	2.9173	3.3062	-0.8872	-0.8854	7.6463	8.9696
64	1.0089	0.0004	0.0476	-0.7472	-0.7413	2.9184	3.3009	-0.8872	-0.8855	7.6509	8.9521
65	1.0087	0.0004	0.0468	-0.7473	-0.7414	2.9195	3.2959	-0.8873	-0.8855	7.6554	8.9353
66	1.0086	0.0004	0.0461	-0.7473	-0.7416	2.9206	3.2910	-0.8873	-0.8856	7.6598	8.9191
67	1.0085	0.0004	0.0454	-0.7474	-0.7417	2.9217	3.2862	-0.8873	-0.8856	7.6641	8.9034
68	1.0083	0.0003	0.0447	-0.7474	-0.7418	2.9227	3.2816	-0.8873	-0.8857	7.6683	8.8882
69	1.0082	0.0003	0.0440	-0.7474	-0.7419	2.9237	3.2772	-0.8874	-0.8857	7.6723	8.8735
70	1.0081	0.0003	0.0434	-0.7475	-0.7421	2.9247	3.2729	-0.8874	-0.8858	7.6763	8.8593
71	1.0080	0.0003	0.0427	-0.7475	-0.7422	2.9256	3.2687	-0.8874	-0.8858	7.6802	8.8456
72	1.0079	0.0003	0.0421	-0.7475	-0.7423	2.9265	3.2647	-0.8874	-0.8859	7.6840	8.8323
73	1.0078	0.0003	0.0415	-0.7476	-0.7424	2.9275	3.2607	-0.8874	-0.8859	7.6877	8.8194
74	1.0077	0.0003	0.0409	-0.7476	-0.7425	2.9283	3.2569	-0.8875	-0.8859	7.6913	8.8068
75	1.0076	0.0003	0.0404	-0.7476	-0.7426	2.9292	3.2532	-0.8875	-0.8860	7.6949	8.7947
76	1.0075	0.0003	0.0398	-0.7477	-0.7427	2.9300	3.2496	-0.8875	-0.8860	7.6983	8.7829
77	1.0074	0.0003	0.0393	-0.7477	-0.7428	2.9309	3.2461	-0.8875	-0.8861	7.7017	8.7715
78	1.0073	0.0003	0.0388	-0.7477	-0.7429	2.9317	3.2427	-0.8875	-0.8861	7.7050	8.7604
79	1.0072	0.0003	0.0383	-0.7478	-0.7430	2.9325	3.2394	-0.8875	-0.8861	7.7083	8.7496
80	1.0071	0.0003	0.0378	-0.7478	-0.7431	2.9332	3.2362	-0.8876	-0.8862	7.7114	8.7391
81	1.0070	0.0002	0.0373	-0.7478	-0.7432	2.9340	3.2331	-0.8876	-0.8862	7.7145	8.7289
82	1.0069	0.0002	0.0368	-0.7478	-0.7432	2.9347	3.2300	-0.8876	-0.8862	7.7176	8.7190
83	1.0069	0.0002	0.0363	-0.7479	-0.7433	2.9354	3.2270	-0.8876	-0.8863	7.7205	8.7093
84	1.0068	0.0002	0.0359	-0.7479	-0.7434	2.9361	3.2241	-0.8876	-0.8863	7.7235	8.6999

Table 6 Continues											
σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	$RCIV = \frac{\sigma_{y_t}^2 - \sigma_{\varepsilon_t}^2}{\sigma_{\varepsilon_t}^2}$								
			IT	**SRT**	**ISRT**	**ST**	**IST**	**CRT**	**ICRT**	**CT**	**ICT**
85	1.0067	0.0002	0.0355	-0.7479	-0.7435	2.9368	3.2213	-0.8876	-0.8863	7.7263	8.6908
86	1.0066	0.0002	0.0350	-0.7479	-0.7436	2.9375	3.2186	-0.8877	-0.8864	7.7291	8.6818
87	1.0065	0.0002	0.0346	-0.7480	-0.7436	2.9381	3.2159	-0.8877	-0.8864	7.7318	8.6731
88	1.0065	0.0002	0.0342	-0.7480	-0.7437	2.9388	3.2132	-0.8877	-0.8864	7.7345	8.6647
89	1.0064	0.0002	0.0338	-0.7480	-0.7438	2.9394	3.2107	-0.8877	-0.8864	7.7372	8.6564
90	1.0063	0.0002	0.0334	-0.7480	-0.7438	2.9400	3.2082	-0.8877	-0.8865	7.7397	8.6483
91	1.0063	0.0002	0.0331	-0.7480	-0.7439	2.9407	3.2058	-0.8877	-0.8865	7.7423	8.6405
92	1.0062	0.0002	0.0327	-0.7481	-0.7440	2.9412	3.2034	-0.8877	-0.8865	7.7448	8.6328
93	1.0061	0.0002	0.0323	-0.7481	-0.7440	2.9418	3.2010	-0.8877	-0.8866	7.7472	8.6253
94	1.0061	0.0002	0.0320	-0.7481	-0.7441	2.9424	3.1988	-0.8878	-0.8866	7.7496	8.6180
95	1.0060	0.0002	0.0316	-0.7481	-0.7442	2.9430	3.1965	-0.8878	-0.8866	7.7519	8.6108
96	1.0059	0.0002	0.0313	-0.7481	-0.7442	2.9435	3.1944	-0.8878	-0.8866	7.7542	8.6038
97	1.0059	0.0002	0.0310	-0.7482	-0.7443	2.9440	3.1922	-0.8878	-0.8866	7.7565	8.5970
98	1.0058	0.0002	0.0306	-0.7482	-0.7444	2.9446	3.1902	-0.8878	-0.8867	7.7587	8.5903
99	1.0058	0.0002	0.0303	-0.7482	-0.7444	2.9451	3.1881	-0.8878	-0.8867	7.7609	8.5838
100	1.0057	0.0002	0.0300	-0.7482	-0.7445	2.9456	3.1861	-0.8878	-0.8867	7.7630	8.5774

Table 7: Mean values, minimum and maximum of the Relative Change in Variance for the Study Transformations

S/N	Transformation	Mean Relative Change in Variance	Minimum	Maximum
1	IT	0.0997	0.03000	0.6512
2	SRT	-0.7453	-0.74821	-0.7317
3	ISRT	-0.7331	-0.74447	-0.6579
4	ST	2.8808	2.72940	2.9456
5	IST	3.7740	3.18600	10.9160
6	CRT	-0.8860	-0.88783	-0.8771
7	ICRT	-0.8824	-0.88671	-0.8556
8	CT	7.5390	7.14470	7.7630
9	ICT	11.8000	8.58000	96.7500

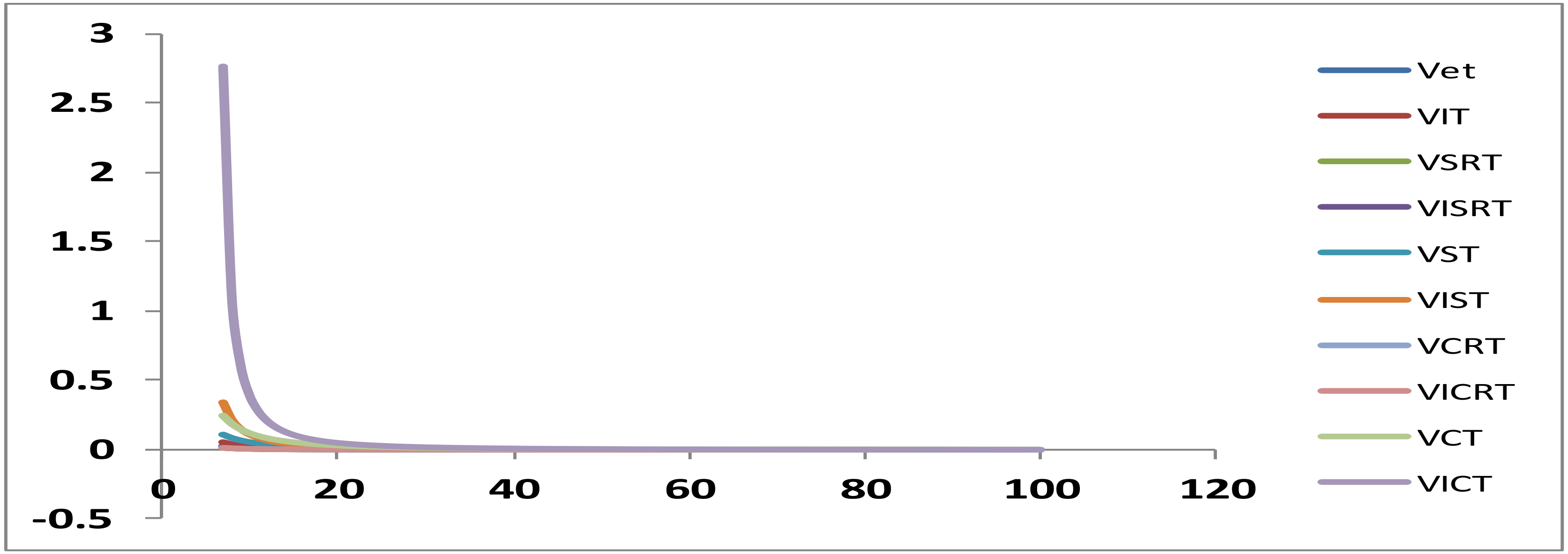

Figure 1: A Plot of the Variances of the Untransformed and the Transformed Distributions against the Shape parameter

where
Vet = Variance of the untransformed Distribution
VIT = Variance of the Inverse Transformed Distribution
VSRT = Variance of the Square Root Transformed Distribution
VISRT = Variance of the Inverse Square Root Transformed Distribution
VST = Variance of the Square Transformed Distribution
VIST = Variance of the Inverse Square Transformed Distribution
VCRT = Variance of the Cube Root Transformed Distribution
VICRT = Variance of the Cube Root Transformed Distribution
VCT = Variance of the Cube Transformed Distribution
VICT = Variance of the Inverse Cube Transformed Distribution

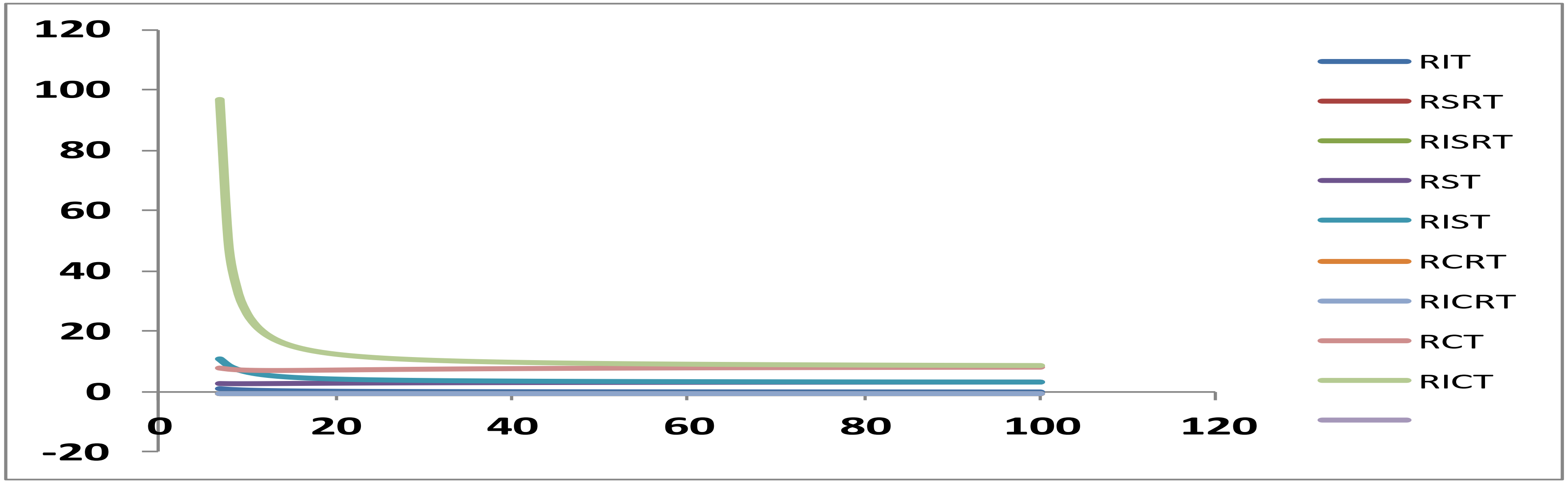

Figure 2: A Plot of the Relative Change in Variance between the Transformed and Untransformed Distribution Against the Shape Parameter

where

RIT = RCIV for the Inverse Transformed Distribution
RSRT = RCIV for the Square Root Transformed Distribution
RISRT = RCIV for the Inverse Square Root Transformed Distribution
RST = RCIV for the Square Transformed Distribution
RIST = RCIV for the Inverse Square Transformed Distribution
RCRT = RCIV for the Cube Root Transformed Distribution
RICRT = RCIV for the Cube Root Transformed Distribution
RCT = RCIV for the Cube Transformed Distribution
RICT = RCIV for the Inverse Cube Transformed Distribution