Power Hamza distribution and its applications to model survival time

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In this paper, a new lifetime model for survival time analysis has been introduced, which is called power Hamza distribution (PHD) that generalizes the Hamza distribution. Some properties of the new distribution, including the probability density function, cumulative distribution function, moments, failure rate function or hazard function and moments were presented. The model provides more flexibility than the Lindley and Hamza distributions in terms of the shape of the density and hazard rate functions. Estimate of the model parameters were obtained via the method of maximum likelihood and applications of the model were made to two real data sets. By using some criteria like Akaike Information criteria (AIC) and Bayesian Information Criteria (BIC) and other statistic, the PHD model provides better fits than other classical lifetime models such as Hamza, exponential and Lindley distributions.

Keywords: power Hamza distribution; moments, reliability, survival function.

1 Introduction

In recent time different life time distributions have been proposed for modelling life time data. Many of these distributions are mixture of exponential and gamma distributions, such as Lindley distribution (Lindley, 1958). Lindley used a mixture of exponential (θ) and gamma (2, θ) distributions to develop Lindley distribution for modelling lifetime data. Studies on the properties of this distribution have shown that it may provide a better fitting than the exponential distribution for some data sets.

Many researchers have developed different generalization and mixture of Lindley distribution (Sankaran, 1970; Ghitany et al., 2008; Zakerzadeh and Dolati, 2009; Nadarajah, et al., 2011; Ghitany et al., 2013 and Aderoju et al., 2017). Another distribution that has been generalized and its mixtures studied recently by different researchers is Sujatha distribution (Shanker, 2016; Shanker and Hagos, 2016; Aderoju, 2020; Tesfay and Shanker, 2019) Recently, a new lifetime distribution called Samade was introduced by Aderoju (2021). The author derived the mathematical properties of Samade distribution and studied its shapes at different values of the parameters. The distribution is a mixture of exponential (θ) and gamma (4, θ) distributions (see Aderoju, 2021).

Asiribo et al. (2019) presented a four-parameter distribution known as the Lomax-Kumaraswamy distribution. The authors studied some properties of the model. The distribution is positively skewed. The implications of the plots for the survival function indicate that the Lomax-Kumaraswamy distribution could be used to model time or age-dependent events, where survival rate decreases with time. The performance of the Lomax-Kumaraswamy distribution was tested by using two real datasets in the literature. The results showed that it can serve as an alternative distribution to modelling positively skewed datasets (Asiribo et al., 2019).

Moreover, Aderoju and Adeniyi (2021) developed a Power Generalized Akash Distribution(PGAD) which extends the generalized Akash distribution introduced by Shanker

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et al. (2018). The PGAD was inspired by the wide use of the Akash distribution and its generalized form in various applied areas. Some structural characteristics of the PGAD were studied and the parameters of the model were obtained via the maximum likelihood estimation method. The flexibility of the distribution was illustrated by its application to two real datasets. Using BIC, AIC and -2Loglikelihood, the authors observed that the PGAD is more effective than Topp-Leone Lomax (TLLo), Generalized Akash (GA), Power Pranav (PP) and Power Transformed Power Inverse Lindley (APTPIL) distributions in modelling real lifetime data.

Aijaz et al. (2020) proposed a new lifetime distribution called Hamza and derived its mathematical properties too. The Hamza distribution is a mixture of exponential (θ) and gamma (7, θ) distributions with combining proportion, $p = \frac{\alpha\theta^5}{120+\alpha\theta^5}$. The probability density function of the distribution was derived as follows:

$P(z|\alpha,\theta) = pg_1(z;\theta) + (1-p)g_2(z;7,\theta)$, where

$$p = \frac{\alpha\theta^5}{120+\alpha\theta^5},$$

$$g_1(z;\theta) = \theta e^{-\theta z} \qquad z > 0,$$

$$g_2(z;7,\theta) = \frac{\theta^7 z^6 e^{-\theta z}}{720}, \qquad z > 0.$$

Hence, the Hamza distribution is

$$P(z|\alpha,\theta) = \begin{cases} \frac{\theta^6}{(120+\alpha\theta^5)}\left(\alpha + \frac{\theta}{6}z^6\right)e^{-\theta z}, & for\ z,\alpha,\theta > 0 \\ 0, & elsewhere \end{cases} \tag{1}$$

The cumulative distribution function (cdf) of (1) was given as:

$$F(z) = 1 - \left[1 + \frac{z\theta(z^5\theta^5 + 6z^4\theta^4 + 30z^3\theta^3 + 120z^2\theta^2 + 360z\theta + 720}{6(120+\alpha\theta^5)}\right]e^{-\theta z} \tag{2}$$

The authors compared the goodness-of-fit of the distribution with some selected one-parameter lifetime distributions and found it to be better than the models.

The objective of this paper is to develop a generalization of Hamza distribution and study its properties in relation to the existing life time related models. We introduced the proposed distribution in Section 2 and studied its mathematical properties in Section 3. The corresponding Maximum Likelihood estimates of the parameters are discussed in Section 4. The application to real life examples and concluding remarks are discussed in Section 5 and 6 respectively.

2 Materials and Methods

In this section, we present the new three-parameter distribution called Power Hamza distribution (PHD) by introducing a shape parameter to the Hamza probability density function. Aijaz et al. (2020) studied in great detail the Hamza distribution and its application. However, there are situations in which the Hamza distribution may not be suitable from a theoretical or applied point of view. So, to obtain a more flexible distribution, we introduce here a new extension of the Hamza distribution by considering the power transformation $X = Z^{\omega^{-1}}$. The pdf of the X can be readily obtained to be

$$f(x|\alpha,\theta,\omega) = pg_1(x;\theta) + (1-p)g_2(x;7,\theta),$$

where

$$p = \frac{\alpha\theta^5}{120 + \alpha\theta^5},$$

$$g_1(x;\theta) = \omega\theta x^{\omega-1}e^{-\theta x^{\omega}}, \qquad x > 0$$

$$g_2(x;7,\theta) = \frac{\omega\theta^7 x^{6\omega-1}e^{-\theta x^{\omega}}}{720}, \qquad x > 0$$

$$f(x|\alpha,\theta,\omega) = \begin{cases} \frac{\omega\theta^6 x^{\omega-1}}{(120 + \alpha\theta^5)}\left(\alpha + \frac{\theta}{6}x^{6\omega}\right)e^{-\theta x^{\omega}}, & for\ x,\alpha,\theta,\omega > 0 \\ 0, & elsewhere \end{cases} \qquad (3)$$

Obviously the Power Hamza distribution is a two-component mixture of Weibull distribution (with shape $\omega$and scale θ), and a generalized gamma distribution (with shape parameters 7, ω and scale θ), with mixing proportion $p = \frac{\alpha\theta^5}{120+\alpha\theta^5}$. The behaviour of $f(x|\alpha,\theta,\omega)$at $x = 0$ and $x = \infty$, respectively, are given by

$$f(0|\alpha,\theta,\omega) = \begin{cases} \infty, & if\ \omega < 1 \\ \frac{\alpha\theta^6}{(120 + \alpha\theta^5)}, & if\ \omega = 1 \\ 0, & if\ \omega > 1 \end{cases}$$

and

$$f(\infty|\alpha,\theta,\omega) = 0.$$

The corresponding cumulative distribution function (cdf) of (3) is obtained as:

$$F(x) = \int_0^x f(t|\alpha,\theta,\omega)\,dt$$

$$= \int_0^x \frac{\omega\theta^6 t^{\omega-1}}{(120 + \alpha\theta^5)}\left(\alpha + \frac{\theta}{6}t^{6\omega}\right)e^{-\theta t^{\omega}}dt$$

$$F(x) = 1 - \left[1 + \frac{x^{\omega}\theta(x^{5\omega}\theta^5 + 6x^{4\omega}\theta^4 + 30x^{3\omega}\theta^3 + 120x^{2\omega}\theta^2 + 360x^{\omega}\theta + 720)}{6(120 + \alpha\theta^5)}\right]e^{-\theta x^{\omega}} \qquad (4)$$

The graphical representation of the density function and cdf of the PHD for some fixed values of the parameters is shown in Figure 1 and Figure 2, respectively.

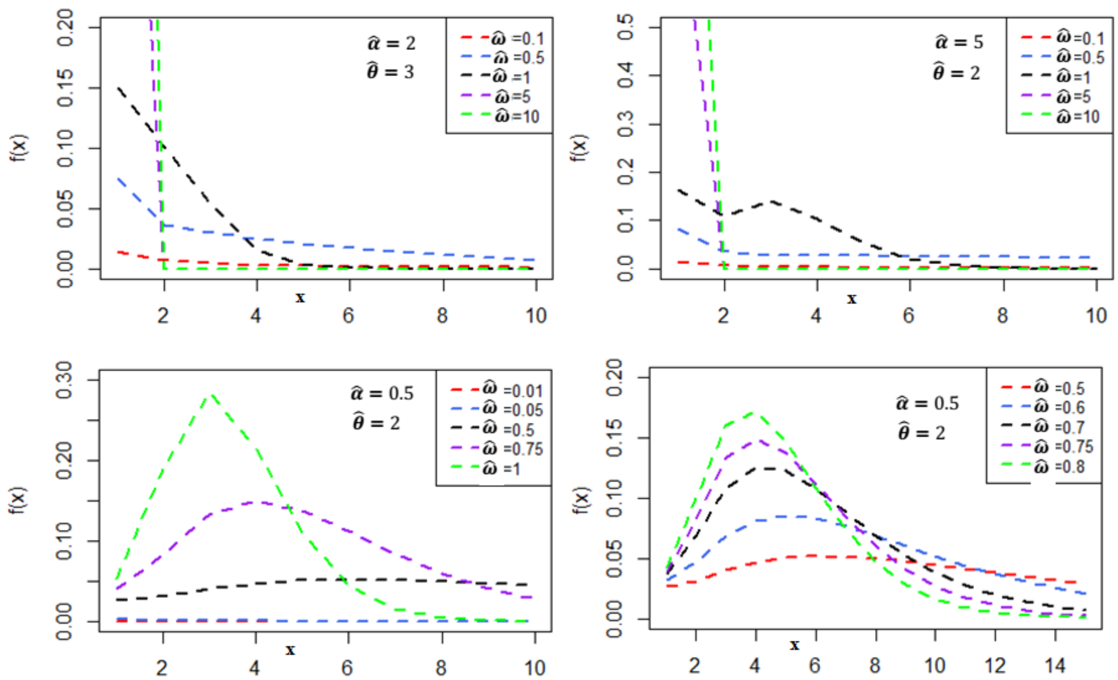

Figure 1: Density Function of the PHD at Different Values of the Parameters

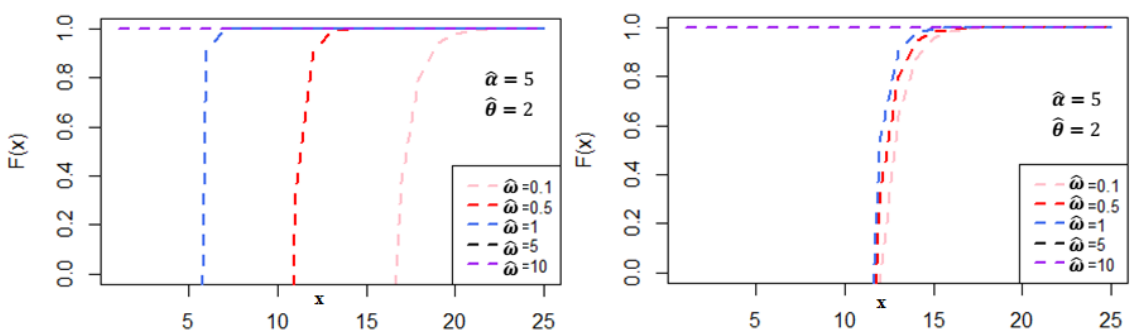

Figure 2: cdf Plot of the PHD at Different Values of the Parameters

3 Statistical Properties of Power Hamza distribution

In this section, some of the Statistical properties of the Power Hamza distribution are presented. The properties include the moments, reliability analysis and the order statistics.

3.1 Moments

Suppose a random variable X follows power Hamza distribution, that is $X \sim PH(\alpha, \theta, \omega)$, then, the r^{th} factorial moment about origin is given by:

$$E(X^r) = \mu_r' = \int_0^\infty x^r f(x) dx$$

$$= \int_0^\infty x^r \frac{\omega \theta^6 x^{\omega-1}}{(120 + \alpha\theta^5)} \left(\alpha + \frac{\theta}{6} x^{6\omega}\right) e^{-\theta x^\omega} dx \qquad (5)$$

Hence, the first four factorial moments are obtained as

$$\mu_1' = \frac{\theta^{-1/\omega}\left(\omega\Gamma\left(7+\frac{1}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{1}{\omega}\right)\right)}{6(120+\alpha\theta^5)\omega};$$

$$\mu_2' = \frac{\theta^{-2/\omega}\left(\Gamma\left(7+\frac{2}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{2+\omega}{\omega}\right)\right)}{6(120+\alpha\theta^5)};$$

$$\mu_3' = \frac{\theta^{-3/\omega}\left(\Gamma\left(7+\frac{3}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{3+\omega}{\omega}\right)\right)}{6(120+\alpha\theta^5)};$$

$$\mu_4' = \frac{\theta^{-4/\omega}\left(\Gamma\left(7+\frac{4}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{4+\omega}{\omega}\right)\right)}{6(120+\alpha\theta^5)}.$$

Note that, the variance (σ^2) of the random variable, X, can be obtained as

$\sigma^2 = E(X^2) - [E(X^1)]^2 = \mu_2' - [\mu_1']^2$. Therefore,

$$\sigma^2 = \frac{\theta^{-4/\omega}\left[6(120+\alpha\theta^5)\left(\Gamma\left(7+\frac{2}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{2+\omega}{\omega}\right)\right) - \frac{\left(\omega\Gamma\left(7+\frac{1}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{1}{\omega}\right)\right)^2}{\omega^2}\right]}{36(120+\alpha\theta^5)^2}$$

The corresponding coefficient of variation (CV) and the index of dispersion, γ, of PHD are obtained as

$$CV = \frac{\sigma}{\mu_1}$$

$$= \frac{\sqrt{\omega\theta^{-1/\omega}\left[6(120+\alpha\theta^5)\left(\Gamma\left(7+\frac{2}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{2+\omega}{\omega}\right)\right) - \frac{\left(\omega\Gamma\left(7+\frac{1}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{1}{\omega}\right)\right)^2}{\omega^2}\right]}}{6(120+\alpha\theta^5)\left(\omega\Gamma\left(7+\frac{1}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{1}{\omega}\right)\right)}.$$

$$\gamma = \frac{\sigma^2}{\mu_1}$$

$$= \frac{\omega\theta^{-1/\omega}\left[6(120+\alpha\theta^5)\left(\Gamma\left(7+\frac{2}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{2+\omega}{\omega}\right)\right) - \frac{\left(\omega\Gamma\left(7+\frac{1}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{1}{\omega}\right)\right)^2}{\omega^2}\right]}{6(120+\alpha\theta^5)\left(\omega\Gamma\left(7+\frac{1}{\omega}\right)+6\alpha\theta^5\Gamma\left(\frac{1}{\omega}\right)\right)}.$$

3.2 Reliability analysis

The reliability characteristics of any given pdf is always considered based on the survival function and the hazard rate function of the distribution. Therefore, the survival function, $S(x)$, and the hazard rate function, $h(x)$, were derived as shown below.

3.2.1 *Survival function*

The survival function is generally defined as the probability that an item does not fail prior to some time. It is expressed as

$$S(x) = 1 - F(x).$$

$$S(x) = \left[\frac{(720+720x^{\omega}\theta+360x^{2\omega}\theta^2+120x^{3\omega}\theta^3+30x^{4\omega}\theta^4+6x^{5\omega}\theta^5+6\alpha\theta^5+x^{6\omega}\theta^6)}{6(120+\alpha\theta^5)}\right] e^{-x^{\omega}\theta}.$$

3.2.2 *Hazard rate function*

The hazard rate function can be expressed as the conditional probability of failure given that it has survived to the time. It is given as

$$h(x) = \frac{f(x)}{S(x)}$$

$$h(x) = \frac{x^{\omega-1}\theta^6(6\alpha+x^{6\omega}\theta)\omega}{720+720x^{\omega}\theta+360x^{2\omega}\theta^2+120x^{3\omega}\theta^3+30x^{4\omega}\theta^4+6x^{5\omega}\theta^5+6\alpha\theta^5+x^{6\omega}\theta^6}.$$

Figures 3 and 4 represent the graph of the survival function and hazard rate function of the Power Hamza distribution, respectively, for varying values of the parameters α, θ andω.

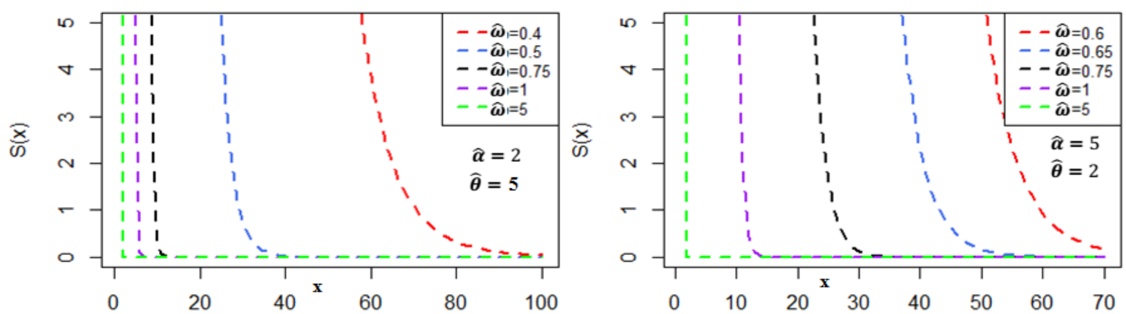

Figure 3: Survival Function of the PHD at Different Values of the Parameters

Plotting the $S(x)$ of the $PH(\alpha, \theta, \omega)$ for different values of the parameters (α, θ, ω) in Figure 3 show that, the shapes of $S(x)$ are decreasing for all selected values of the shape parameters (α, ω) and the scale parameter (θ); this also shows how flexible the behaviour of the $S(x)$.

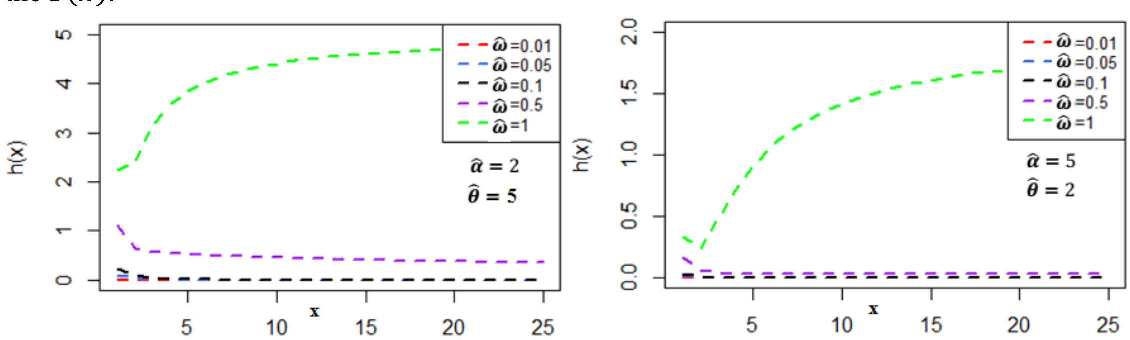

Figure 4: Hazard Rate Function of the PHD at Different Values of the Parameters

The graphs of the hazard rate function of the PHD for different values of the parameters are given in Figure 4. Obviously, the model exhibits both monotone increasing and decreasing failure rate characteristic. It decreases monotonically when $\omega < 1$ and increases monotonically when $\omega \geq 1$.

3.3 Order statistics

Order statistics gives one of the popular fundamental tools for obtaining inference related to reliability data. The largest order and the smallest are denoted as $X_n = \max(X_1, X_2, \dots, X_n)$ and $X_1 = \min(X_1, X_2, \dots, X_n)$, where n is the sample size. Suppose

$X_{(1)} \leq X_{(2)} \leq \ldots \leq X_{(n)}$ are order observations of $X_1, X_2, \ldots, X_n$ taken from the studied distribution then density of the k^{th} order statistic $X_{(k)}$ can be expressed as:

$$f_{X(k)}(x) = \frac{n!}{(k-1)!\,(n-k)!} f(x)[F(x)]^{k-1}[1-F(x)]^{n-k} \quad (6)$$

$$= \frac{n!\,\omega\theta^6 x^{\omega-1}}{(120+\alpha\theta^5)(k-1)!\,(n-k)!}\left(\alpha + \frac{\theta}{6}x^{6\omega}\right)e^{-\theta x^{\omega}}\left[1 - \left[1 + \frac{x^{\omega}\theta(x^{5\omega}\theta^5 + 6x^{4\omega}\theta^4 + 30x^{3\omega}\theta^3 + 120x^{2\omega}\theta^2 + 360x^{\omega}\theta + 720)}{6(120+\alpha\theta^5)}\right]e^{-\theta x^{\omega}}\right]^{k-1}$$

$$\times \left[\left[\frac{(720 + 720x^{\omega}\theta + 360x^{2\omega}\theta^2 + 120x^{3\omega}\theta^3 + 30x^{4\omega}\theta^4 + 6x^{5\omega}\theta^5 + 6\alpha\theta^5 + x^{6\omega}\theta^6)}{6(120+\alpha\theta^5)}\right]e^{-x^{\omega}\theta}\right]^{n-k}$$

The first and n^{th} orders are

$$f_{X(1)}(x) = \frac{n!\omega\theta^6 x^{\omega-1}}{(120+\alpha\theta^5)(n-1)!}\left(\alpha + \frac{\theta}{6}x^{6\omega}\right)e^{-\theta x^{\omega}}\left[\left[\frac{(720+720x^{\omega}\theta+360x^{2\omega}\theta^2+120x^{3\omega}\theta^3+30x^{4\omega}\theta^4+6x^{5\omega}\theta^5+6\alpha\theta^5+x^{6\omega}\theta^6)}{6(120+\alpha\theta^5)}\right]e^{-x^{\omega}\theta}\right]^{n-1}.$$

and

$$f_{X(n)}(x) = \frac{n!\omega\theta^6 x^{\omega-1}}{(120+\alpha\theta^5)(n-1)!}\left(\alpha + \frac{\theta}{6}x^{6\omega}\right)e^{-\theta x^{\omega}}\left[1 - \left[1 + \frac{x^{\omega}\theta(x^{5\omega}\theta^5+6x^{4\omega}\theta^4+30x^{3\omega}\theta^3+120x^{2\omega}\theta^2+360x^{\omega}\theta+720)}{6(120+\alpha\theta^5)}\right]e^{-\theta x^{\omega}}\right]^{n-1}.$$

3.4 Maximum likelihood estimation

Suppose $X_1, X_2, X_3, \ldots, X_n$ is a random sample of size n from the PHD, the maximum likelihood function of parameters can be written as

$$L(\alpha,\theta,\omega) = \prod_{i=1}^{n}\frac{\omega\theta^6 {x_i}^{\omega-1}}{(120+\alpha\theta^5)}\left(\alpha + \frac{\theta}{6}{x_i}^{6\omega}\right)e^{-\theta {x_i}^{\omega}},$$

and the log-likelihood function is

$$lnL = \ell = lnL(\alpha,\theta,\omega) = nln(\omega) + 6nln(\theta) + (\omega-1)\sum_{i=1}^{n} lnx_i - nln(120+\alpha\theta^5) + \sum_{i=1}^{n}\ln\left(\alpha + \frac{\theta}{6}{x_i}^{6\omega}\right) - \theta\sum_{i=1}^{n} x_i^{\omega}.$$

Hence,

$$\frac{\partial lnL}{\partial\alpha} = -\frac{n\theta^5}{120+\alpha\theta^5} + \sum_{i=1}^{n}\left(\alpha + \frac{\theta}{6}{x_i}^{6\omega}\right)^{-1} = 0 \quad (7)$$

$$\frac{\partial lnL}{\partial\theta} = \frac{6n}{\theta} - \frac{5n\alpha\theta^4}{120+\alpha\theta^5} + \sum_{i=1}^{n}\frac{1}{6}{x_i}^{6\omega}\left(\alpha + \frac{1}{6}{x_i}^{6\omega}\right)^{-1} - \sum_{i=1}^{n} x_i^{\omega} = 0 \quad (8)$$

$$\frac{\partial lnL}{\partial\omega} = \frac{n}{\omega} + \sum_{i=1}^{n}\ln(x_i) + \sum_{i=1}^{n}\frac{\theta x_i^{6\omega}\ln(x_i)}{\alpha + \frac{\theta}{6}{x_i}^{6\omega}} - \theta\sum_{i=1}^{n} x_i^{\omega}\ln(x_i) = 0 \quad (9)$$

Differentiating the $logL(\alpha,\theta,\omega)$ partially with respect to associated parameters and equating the zero as shown in (7), (8) and (9) should provide solution algebraically; the Maximum Likelihood Estimates (MLEs), $\hat{\alpha}$, $\hat{\theta}$ and $\hat{\omega}$, of α, θ and ω are solutions of the equations. Obviously, it is very difficult to solve the system of nonlinear equations; hence, we computed the MLEs numerically using the *nloptr* package and *bobyqa* function in R software (R Core Team, 2020).

4 Application

In this section, we provide two applications of the PHD including the estimation of the parameters through the method of maximum likelihood. We as well compared its performances with Exponential, Power Pranav, Power quasi Lindley, and Hamza distributions (that is, ED, PPD, PQLD and HD respectively). The probability distribution functions of these competing lifetime distributions are detailed in Table 1.

Table 1: Distributions Considered

Name of distribution	Probability density functions	Author(s)
Exponential Distribution (ED)	$f(x)=\theta e^{-\theta x}$	
Lindley Distribution (LD)	$f(x)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x}$	Lindley (1958)
Hamza Distribution (HD)	$f(x)=\frac{\theta^6}{(120+\alpha\theta^5)}\left(\alpha+\frac{\theta}{6}x^6\right)e^{-\theta x}$	Aijaz et al., (2020)
Power Pranav Distribution (PPD)	$f(x)=\frac{\omega\theta^4x^{\omega-1}}{(2+\theta^4)}(\theta+x^{3\omega})e^{-\theta x^{\omega}}$	Shukla (2019)
Power Quasi Lindley Distribution (PQLD)	$f(x)=\frac{\omega\theta x^{\omega-1}}{(1+\alpha)}(\alpha+\theta x^{\omega})e^{-\theta x^{\omega}}$	Alkami (2015)
Power Hamza Distribution (PHD)	$f(x)=\frac{\omega\theta^6x^{\omega-1}}{(120+\alpha\theta^5)}\left(\alpha+\frac{\theta}{6}x^{6\omega}\right)e^{-\theta x^{\omega}}$	This paper

Data set 1: The data set of waiting times (in minutes) before service of 100 bank customers as discussed by Ghitany et al., (2008). The waiting times (in minutes) are as follows:

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6,2.7,2.9, 3.1, 3.2,3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3,4.3,4.4,4.4, 4.6, 4.7,4.7, 4.8, 4.9, 4.9, 5.0, 5.3, 5.5, 5.7,5.7,6.1,6.2,6.2,6.2,6.3, 6.7,6.9,7.1,7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6,8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6, 9.7, 9.8, 10.7, 10.9, 11.0,11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4, 12.5, 12.9, 13.0, 13.1, 13.3, 13.6,13.7, 13.9,14.1,15.4,15.4,17.3,17.3,18.1,18.2,18.4,18.9, 19.0, 19.9,20.6,21.3, 21.4, 21.9, 23.0, 27.0, 31.6,33.1,38.5.

Table 2: Statistics of the Fitted Distributions of Data Set 1

Models	Parameter estimates	-2logL	AIC	AICC	BIC	Ranks
PHD	$\hat{\alpha}=0.9301$ $\hat{\theta}=1.4029$ $\hat{\omega}=0.7058$	634.352	640.352	640.602	643.5624	1
HD	$\hat{\alpha}=680.35$ $\hat{\theta}=0.5621$	694.2228	698.228	698.3465	703.4332	5
PPD	$\hat{\alpha}=0.7548$ $\hat{\theta}=0.7176$	637.3822	641.3822	641.3822	646.5925	2
PQLD	$\hat{\alpha}=240.274$ $\hat{\theta}=0.0307$ $\hat{\omega}=1.4574$	637.4618	643.4618	643.7118	646.6721	3
ED	$\hat{\theta}=0.1012$	658.04	660.04	660.13	662.65	4

The best goodness-of-fit results statistics for -2logL, AIC, AICC and BIC are in bold font.

Table 2 presents the summary of the results of fitness of the model and the competing models to data on "the waiting times (in minutes) before service of 100 bank customers". Clearly, the PHD fits the data better than the competing models having the least AIC (640.352).

Data set 2: The second data set considered here is the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal et al. (1960) and Jamal et al., (2019). The data are as follows:

0.1, 0.33, 0.44, 0.56, 0.59, 0.72, 0.74, 0.77, 0.92, 0.93, 0.96, 1.0, 1.0, 1.02, 1.05, 1.07, 1.07, 1.08, 1.08, 1.08, 1.09, 1.12, 1.13, 1.15,1.16, 1.2,1.21, 1.22, 1.22, 1.24, 1.3, 1.34, 1.36, 1.39, 1.44, 1.46, 1.53, 1.59, 1.6,1.63, 1.63, 1.68, 1.71, 1.72, 1.76, 1.83, 1.95, 1.96, 1.97, 2.02, 2.13, 2.15, 2.16, 2.22, 2.3, 2.31, 2.4, 2.45, 2.51, 2.53, 2.54, 2.54, 2.78, 2.93, 3.27, 3.42, 3.47, 3.61, 4.02, 4.32, 4.58, 5.55.

Table 3: ML Statistics of the Fitted Distributions of Data Set 2

Models	Parameter estimates	-2logL	AIC	AICC	BIC	Ranks
PHD	$\hat{\alpha} = 7.6 \times 10^{-4}$ $\hat{\theta} = 4.7723$ $\hat{\omega} = 0.7048$	**186.3988**	**192.3988**	**192.3988**	**194.9522**	1
HD	$\hat{\alpha} = 0.0076$ $\hat{\theta} = 3.7971$	204.1182	208.1182	208.2921	212.6715	3
PPD	$\hat{\alpha} = 1.3489$ $\hat{\theta} = 1.3002$	206.4012	210.4012	210.5751	214.9545	4
PQLD	$\hat{\alpha} = 0.0221$ $\hat{\theta} = 0.9032$ $\hat{\omega} = 1.2802$	188.7954	194.7954	195.1483	197.3487	2
ED	$\hat{\theta} = 0.5655$	226.074	228.0741	228.1312	234.6274	5

The best goodness-of-fit has been shown by marking -2logL, AIC, AICC and BIC Statistics in bold

Table 3 presents the summary of the results fitness of the model and the competing models to the data on "survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli". Obviously, the PHD also fits the data better than the competing models having the least AIC (192.3988).

5 Conclusion

In this paper, we proposed a new life time distribution which generalizes the Hamza distribution. We have presented and studied the properties of the power Hamza distribution (PHD), which has Hamza and exponential distribution and its special cases. We studied some of its mathematical and statistical properties, including the pdf, cdf, moments, failure rate function or hazard function and the reverse hazard function. Reliability and order statistics for PHD were also derived. Estimates of the model parameters were obtained via the method of maximum likelihood and applications of the model were made to two real data sets and presented in Table 2 and Table 3. It was observed that the PHD model provides better fits than other common lifetime models, considering the values of the LogLik, AIC and BIC.

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