Selecting superior GARCH model with backtesting approach in First Bank of Nigeria stock returns

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The evaluation of financial risk models or backtesting is an important part of the internal model's approach to market risk management. Unfortunately, the backtesting approach is not popular among financial analysts in Nigeria. Backtesting is a statistical procedure where actual profits and losses are systematically compared to corresponding VaR estimates. This study investigates the volatility in daily stock returns of First Bank of Nigeria using nine variants of GARCH models: sGARCH (1,1), girGARCH (1,1), eGARCH(1,1), iGARCH(1,1), apARCH(1,1), TGARCH(1,1), NGARCH(1,1), NAGARCH (1,1), and AVGARCH (1,1) along with value-at-risk estimation through backtesting approach using student t and skewed student t innovations. We use daily data for First Bank of Nigeria returns for the period, January 2, 2001 to May 8, 2017 obtained from a secondary source. Most of the models were promising in terms of information criteria and ARCH test after estimation but failed the backtesting analysis. With the backtesting approach, eGARCH (1,1) model with student t distribution emerged as the superior GARCH model among the competing GARCH models for modeling First Bank returns in Nigeria. This study recommends that backtesting approach can enhance modeling selection and reliable inferences among financial analysts and practitioners.

Keywords: GARCH; information criteria; value-at-risk; stock; returns

1. Introduction

Generalized autoregressive conditional heteroscedasticity (GARCH) models are various extensions and improvements on the autoregressive conditional heteroscedasticity (ARCH) model developed by Sir Robert F. Engle in 1982 to handle time-varying volatility and uncertainty inherent in financial time series, which model was the first to assume that volatility is not constant (Lawrence, 2013; Atoi, 2014; Grek, 2014; Emenogu and Adenomon, 2018). Some of the various variants of ARCH and GARCH models include: standard generalized autoregressive conditional heteroscedasticity (sGARCH), Glosten-Jagannathan-Ronkle generalized autoregressive conditional heteroscedasticity (gjrGARCH), exponential generalized autoregressive conditional heteroscedasticity (eGARCH), asymmetric power autoregressive conditional heteroscedasticity (apARCH), integrated generalized autoregressive conditional heteroscedasticity (iGARCH), threshold generalized autoregressive conditional heteroscedasticity (TGARCH), nonlinear generalized autoregressive conditional heteroscedasticity (NGARCH), nonlinear asymmetric generalized autoregressive conditional heteroscedasticity (NAGARCH), absolute value generalized autoregressive conditional heteroscedasticity (AVGARCH) models, etc. (Ali, 2013; Atoi, 2014 and Emenogu *et al*., 2018).

In this study, we investigate the volatility in daily stock returns of First Bank of Nigeria using nine variants of GARCH family models, namely: sGARCH(1,1), gjrGARCH(1,1),

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eGARCH(1,1), apARCH(1,1), iGARCH(1,1), TGARCH(1,1), NGARCH(1,1), NAGARCH(1,1) and AVGARCH(1,1) along with value-at-risk (VaR) estimation through backtesting approach using student *t* and skewed student *t* innovations with a view to selecting the superior model from among the competing nine variants of GARCH models listed above. The use of backtesting technique here to select the superior model is informed by Emenogu (2019) which asserted that using information criteria alone for model selection could sometimes give misleading results.

Backtesting or Financial risk model evaluation is a statistical procedure where actual profits and losses are systematically compared to correspond to corresponding VaR estimates (Nieppola, 2009 and Emenogu *et al.*, 2018). Backtesting approach is very useful in GARCH model selection, but it is not yet popular among financial analysts in Nigeria. However, Emenogu *et al.* (2019) found that Summinga-Sonagadu and Narsoo (2019) employed three backtesting techniques namely: Kupiec's test, a duration-based test and an asymmetric VaR loss function on intraday of 1-min EUR/USD exchange rate returns, and found that VaR prediction of the MC-GARCH model performed better using the asymmetric loss function.

The data used in this study are daily stock returns of First Bank of Nigeria for the period, January 2, to May 8, 2017, obtained from a secondary source. Brief History of First Bank Nigeria as extracted from Wikipedia.org (2020/08/21) has it that the First Bank of Nigeria commenced business in 1894 in what was then the British colony of Nigeria, as the Bank of British West Africa to serve the interest of British shipping and trading agencies in Nigeria. The bank primarily financed foreign trade, but did little lending to indigenous Nigerians, who had little to offer as collateral for loans. But after Nigeria's independence in 1960, the Bank began to extend more credit to indigenous Nigerians. Consequently, Nigerian citizens began to trust the bank among other British Banks, resulting in more citizens patronizing the new Bank of West Africa. When in 1965, the Bank was acquired by Standard Bank, its name was changed to Standard Bank of West Africa, and to Standard Bank of Nigeria in 1969 when it incorporated its Nigerian operations. In 1971, Standard Bank of Nigeria listed its shares on the Nigerian Stock Exchange and placed 13% of its share capital with Nigerian investors. However, the name of the Bank was changed to First Bank of Nigeria Limited in 1979 following the loss of majority control by Standard Chartered Bank which reduced its stake in Standard Bank Nigeria to 38%. In 1991, the Bank changed its name to First Bank of Nigeria Plc following listing on The Nigerian Stock Exchange. In 2012, the Bank changed its name again to First Bank of Nigeria Limited as part of a restructuring resulting in FBN Holdings Plc, having detached its commercial business from other businesses in the First Bank Group, in line with the requirements of the Central Bank of Nigeria. Currently, First Bank has branches in London and in Johannesburg, South Africa.

2. Literature Review

2.1 Backtesting

Financial risk model evaluation or backtesting is an important part of the internal model's approach to market risk management as put out by Basle Committee on Banking Supervision (Christoffersen and Pelletier, 2004). It should be recalled that backtesting, according to Nieppola (2009) and Emenogu *et al.* (2018), is a statistical procedure that enables systematic comparison of actual profits and losses to corresponding VaR estimates. Summinga-Sonagadu and Narsoo (2019) utilized three backtesting procedures in their analysis of EURO/USD exchange rate, namely: Kupiec's test, a duration-based test and an asymmetric VaR loss function on intraday of 1-min EUR/USD exchange rate returns, and found that VaR prediction of the MC-GARCH model performed better using the asymmetric loss function.

Emenogu *et al.* (2020) conducted duration-based tests of independence. Under the null hypothesis that the risk model was correctly specified, the no-hit duration should have no memory of 1/p days. The duration-based tests of independence conducted reveal that the models were correctly specified, meaning that the probability of an exception on any day did not depend on the outcome of the previous day. Also, Emenogu *et al.* (2019) applied backtesting in selecting from among four competing ARMA-GARCH models using the student *t* distribution and skewed student *t* distribution, and found that ARMA(1,1)-eGARCH(2,2) was better than the other models. But Painter *et al.* (2017) dealt on the methods of obtaining exceedance-probability of stream flows for streams in Kansas.

This study adopted Backtesting techniques of Christoffrsen and Pelleitier (2004). The test was implemented in R software using rugarch package and this test considered both the unconditional (Kupiec) and conditional (Christoffersen) coverage tests for the correct number of exceedances: see details in Christoffersen (1998) and Christoffersen *et al*. (2001).

2.2 Value-at-risk (VaR)

Value-at-risk is defined as the maximum a given amount of currency or price of stock is expected to lose over a given time horizon, at a pre-defined confidence level (Best, 1998; Bali and Cakici, 2004). It is a statistical measure of the riskiness of financial entities or portfolio of assets (Corkalo, 2011). Okpara (2015) studied risk analysis of the Nigerian stock market using the VaR approach. Based on Akaike Information Criteria, the study suggested that EGARCH model with student *t* innovation distribution could furnish more accurate estimate of VaR, and applying the likelihood ratio tests of proportional failure rates to VaR derived from the EGARCH model, concluded that investors and portfolio managers in the Nigerian stock market have long trading position. Corkalo (2011) conducted a comparative study of the main approaches of calculating VaR similar to that by van den Goorbergh and Vlaar (1999). The study implemented variance-covariance, historical simulation and bootstrapping approach on stock portfolio and presented results using histogram. Based on the results, it recommended that investor and risk managers should look at composition of its portfolio and then choose appropriate method to calculate VaR.

Tay *et al*. (2019) investigated the efficiency of the Value-at-Risk (VaR) backtesting in model selection from different types of GARCH models with skewed and non-skewed innovation distributions. The study implemented both simulation and real-life data application (NASDAQ Index). The study revealed that AIC and VaRbacktesting approaches were able to select the correct model with their corresponding innovation distributions. Emenogu *et al.* (2020) implemented VaR calculation in assessing model performance on log returns and cleansed log returns of stock of Total Nigeria Plc, and found that: for normal innovation, eGARCH and sGARCH performed best while for student *t* innovation, NGARCH performed best.

Notwithstanding the fact that VaR computation has been asserted to be a better risk measure for estimating portfolio risk than any risk measure whose accuracy cannot be verified, (Tripathi and Aggarwal, 2008), there are limited literatures on the application of Value-at-Risk on stocks in Nigeria; for instance, Eyisi and Oleka (2014) did not apply VaR in their study of management and portfolio analysis in the capital market in Nigeria but based their risk measurement on the size of the difference between the actual returns (R) and the expected returns $\sum(R)$.

3. Materials and Methods

3.1 Data

The data used in this study are daily stock price for First Bank of Nigeria from January 2, 2001 to May 8, 2017, collected on 8th May, 2017 from the website of Cashcraft Asset Management Ltd, www.cashcraft.com, a total of 4017 observations.

3.2 Model Specification

We focus this study on the GARCH models that are robust for forecasting the volatility of financial time series data; so GARCH model and some of its extensions are presented in this section.

3.2.1 (ARCH) Family model

Atoi (2014) stated that every ARCH or GARCH family model requires two distinct specifications of the mean and the variance equations. The mean equation for a conditional heteroscedasticity in a return series, y_t, is given by

$$y_t = E_{t-1}(y_t) + \varepsilon_t \quad (1)$$

where

$$\varepsilon_t = \phi_t \sigma_t$$

The mean equation also applies to other GARCH family models, $E_{t-1}(.)$ is the expected value conditional on information available at time *t-1*, while ε_t is the error generated from the mean equation at time *t* and $\phi_t$is the sequence of independent and identically distributed random variables with zero mean and unit variance. The variance equation for an ARCH(p) model is given by

$$\sigma_t^2 = \omega + \alpha_1 a_{t-1}^2 + \cdots + \alpha_p a_{t-p}^2. \quad (2)$$

It can be seen in the equation that large values of the innovation of asset returns have bigger impact on the conditional variance because they are squared, which means that a large shock tends to follow another large shock and that is the same way the clusters of the volatility behave. So, the ARCH(p) model becomes

$$a_t = \sigma_t \varepsilon_t, \qquad \sigma_t^2 = \omega + \alpha_1 a_{t-1}^2, \tag{3}$$

where $\varepsilon_t \sim N(0,1), \omega > 0$ and $\alpha_i \geq 0$ for $i > 0$. In practice, ε_t is assumed to follow the standard normal or a standardized student *t* distribution or a generalized error distribution (Tsay, 2005).

3.2.2 Asymmetric Power ARCH

The importance of apARCH model, according to Rossi (2004), is that it forms the basis for deriving the GARCH family of models; the model is given as follows.

$$r = \mu + a_t,$$
$$\varepsilon_t = \sigma_t \varepsilon_t,$$
$$\varepsilon_t \sim N(0,1)$$
$$\sigma_t^\delta = \omega + \sum_{i=1}^{p} \alpha_i (|a_{t-i}| - \gamma_i a_{t-i})^\delta + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^\delta, \tag{4}$$

where

$$\omega > 0, \qquad \delta \geq 0,$$
$$\alpha_i \geq 0, \qquad i = 1, 2, \cdots, p$$
$$-1 < \gamma_i < 1, \quad i = 1, 2, \cdots, p$$
$$\beta_j > 0, \qquad j = 1, 2, \cdots, q,$$

This model imposes a Box-Cox transformation of the conditional standard deviation process and the asymmetric absolute residuals. The leverage effect is the asymmetric response of volatility to positive and negative "shocks".

3.2.3 Standard GARCH(1,1) model

The mathematical model for the GARCH(p, q) model is obtained from equation (4) by letting $\delta = 2$ and $\gamma_i = 0, i = 1, \cdots, p$ to be

$$a_t = \sigma_t \varepsilon_t,$$
$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{5}$$

where $a_t = r_t - \mu_t$ (r_t, is the continuously compounded log return series), and $\varepsilon_t \sim$N(0, 1), the parameter α_i is the ARCH parameter and β_j is the GARCH parameter, and $\omega > 0$, $\alpha_i \geq 0$, $\beta_j \geq 0$, and $\sum_{i=1}^{\max(p,q)} (\alpha_i + \beta_i) < 1$, (Rossi, 2004; Tsay, 2005 and Jiang, 2012). The restriction on ARCH and GARCH parameters (α_i, β_i) suggests that the volatility (a_i) is finite and that the conditional standard deviation (σ_i) increases. It can be observed that if q = 0, then the model GARCH parameter (β_j) becomes extinct and what is left is an ARCH(p) model. But when p =1 and q =1, we have GARCH(1, 1) model given by

$$a_t = \sigma_t \varepsilon_t,$$
$$\sigma_t^2 = \omega + \alpha_i a_{t-i}^2 + \beta_j \sigma_{t-j}^2. \tag{6}$$

3.2.4 GJR-GARCH(1,1) Model

The Glosten-Jagannathan-Runkle GARCH (GJRGARCH) model, which is a model that attempts to address volatility clustering in an innovation process, is obtained by letting $\delta = 2$. When $\delta = 2\ and\ 0 \leq \gamma_i < 1$,

$$\sigma_t^2 = \omega + \sum_{i=1}^{p} \alpha_i (|\varepsilon_{t-i}| - \gamma_i \varepsilon_{t-i})^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \quad (7)$$

and it can be shown that for $p = 1$ and $q - 1$, we shall have GJR-GARCH(1,1) Model given by

$$\sigma_t^2 = \omega + \alpha\varepsilon_t^2 + \gamma S_i \varepsilon_t^2 + \beta\sigma_{t-1}^2 \quad (8)$$

3.2.5 EGARCH model

The eGARCH model proposed by Nelson (1991) was to overcome some weaknesses of the GARCH model in handling financial time series pointed out by Enocksson and Skoog (2012). The eGARCH was particularly to allow for asymmetric effects between positive and negative asset returns, the weighted innovation below was considered

$$g(\varepsilon_t) = \theta\varepsilon_t + \gamma[|\varepsilon_t| - E(|\varepsilon_t|)], \quad (9)$$

where $\theta$and γ are real constants. Both ε_t and $|\varepsilon_t| - E(|\varepsilon_t|)$ are zero-mean iid sequences with continuous distributions. Therefore, $E[g(\varepsilon_t)] = 0$. The asymmetry of $g(\varepsilon_t)$ can easily be seen by rewriting it as

$$g(\varepsilon_t) = \begin{cases} (\theta + \gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) \text{if} \varepsilon_t \geq 0, \\ (\theta - \gamma)\varepsilon_t - \gamma E(|\varepsilon_t|) if \varepsilon_t < 0. \end{cases} \quad (10)$$

An eGARCH(*m*, *s*) model, according to Tsay (2005), Dhamija and Bhalla (2010), Jiang (2012), Ali (2013) and Grek (2014), can be written as:

$$a_t = \sigma_t \varepsilon_t, \qquad ln(\sigma_t^2) = \omega + \sum_{i=1}^{s} \alpha_i \frac{|a_{t-i}| +}{\sigma_{t-i}} + \sum_{j=1}^{m} \beta_j ln(\sigma_{t-i}^2), \quad (11)$$

which specifically results in eGARCH(1,1) being written as $a_t = \sigma_t \varepsilon_t$,

$$ln(\sigma_t^2) = \omega + \alpha([|a_{t-1}| - E(|a_{t-1}|)]) + \theta a_{t-1} + \beta \ln(\sigma_{t-1}^2) \quad (12)$$

where $|a_{t-1}| - E(|a_{t-1}|)$ are independent and identically distributed and have mean, zero. When the eGARCH model has a Gaussian distribution of error term, then $E(|\varepsilon_t|) = \sqrt{2/\pi}$, which gives

$$ln(\sigma_t^2) = \omega + \alpha\left(\left[|a_{t-1}| - \left(\sqrt{2/\pi}\right)\right]\right) + \theta a_{t-1} + \beta \ln(\sigma_{t-1}^2) \quad (13)$$

3.2.6 iGARCH(1,1) model

The integrated GARCH (iGARCH) models are unit-root GARCH models. The iGARCH(1,1) model is specified in Tsay (2005) and Grek (2014) as

$$a_t = \sigma_t \varepsilon_t;$$
$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + (1 - \beta_1) a_{t-1}^2 \quad (14)$$

where $\varepsilon_t \sim N(0,1)$, and $0 < \beta_1 < 1$.

The model is also an exponential smoothing model for the $\{a_t^2\}$ series. To see this, rewrite the model as

$$\begin{aligned}\sigma_t^2 &= (1-\beta_1)a_{t-1}^2 + \beta_1\sigma_{t-1}^2 \\ &= (1-\beta_1)a_{t-1}^2 + \beta_1[(1-\beta_1)a_{t-2}^2 + \beta_1\sigma_{t-2}^2] \\ &= (1-\beta_1)a_{t-1}^2 + (1-\beta_1)\beta_1 a_{t-2}^2 + \beta_1^2\sigma_{t-2}^2. \end{aligned} \tag{15}$$

By repeated substitutions, we have

$$\sigma_t^2 = (1-\beta_1)(a_{t-1}^2 + \beta_1 a_{t-2}^2 + \beta_1^2 a_{t-3}^3 + \cdots), \tag{16}$$

which is the well-known exponential smoothing formation with $\beta_1$being the discounting factor (Tsay, 2005).

3.2.7 TGARCH(1,1) model

The Threshold GARCH model is another model useful for handling leverage effects, and a TGARCH(p, q) model is given by the following

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p}(\alpha_i + \gamma_i N_{t-i})^2 a_{t-i}^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2, \tag{17}$$

where $N_{t-i}$is an indicator for negative a_{t-i}. That is,

$$N_{t-i} = \begin{cases} 1 \text{ if } a_{t-i} < 0, \\ 0 \text{ if} a_{t-i} \geq 0, \end{cases}$$

and α_i, γ_i, and β_j, are non-negative parameters satisfying conditions similar to those of GARCH models, (Tsay, 2005). When $p = 1, q = 1$, the TGARCH(1, 1) model becomes:

$$\sigma_t^2 = \omega + (\alpha + \gamma N_{t-1})a_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{18}$$

3.2.8 NGARCH(1,1) model

NGARCH Model has been presented in various different ways in literature by the following scholars: Hsieh and Ritchken (2005), Lanne and Saikkonen (2005), Malecka (2014) and Kononovicius and Ruseckas (2015). The following model can be shown to represent all the presentations:

$$h_t = \omega + \alpha\varepsilon_t^2 + \gamma\varepsilon_t + \beta h_t \tag{19}$$

where h_t is the conditional variance, and ω, β and α satisfy $\omega > 0$, $\beta \geq 0$ and $\alpha \geq 0$, which can also be written as

$$\sigma_t = \omega + \alpha\varepsilon_t^2 + \gamma\varepsilon_t + \beta\sigma_t \tag{20}$$

3.2.9 Nonlinear (Asymmetric) GARCH, or N(A)GARCH or NAGARCH

NAGARCH plays key role in option pricing with stochastic volatility because it allows you to derive closed-form expressions for European option prices in spite of the rich volatility dynamics. A NAGARCH may be written as

$$\sigma_{t-1}^2 = \omega + \alpha\sigma_t^2(z_t - \delta)^2 + \beta\sigma_t^2 \tag{21}$$

and if $z_t \sim N(0,1)$, z_t is independent of σ_t^2 as σ_t^2 is only a function of past squared returns, it is possible to easily derive the long run, unconditional variance under NGARCH and the assumptions of stationarity,

$$E|\sigma_{t+1}^2| = \bar{\sigma}^2 = \alpha E[\sigma_t^2(z_t-\delta)^2] + \beta E[\sigma_t^2]$$
$$= \omega + \alpha E[\sigma_t^2]E[z_t^2+\delta^2-2\delta z_t] + \beta E[\sigma_t^2]$$
$$= \omega + \alpha\bar{\sigma}^2(1+\delta^2) + \beta\bar{\sigma}^2 \quad (22)$$

where $\bar{\sigma}^2 = E[\sigma_t^2]$ and $E[\sigma_t^2] = E[\sigma_{t+1}^2]$ because of stationarity. Therefore,

$$\bar{\sigma}^2[1-\alpha(1+\delta^2)+\beta] = \omega \Longrightarrow \bar{\sigma}^2 = \frac{\omega}{1-\alpha(1+\delta^2)+\beta} \quad (23)$$

which, according to Nelson (1991), Hall & Yao (2003), Enders (2004), Christoffersen *et al.* (2008) and Engle & Rangel (2008), exists and positive if and only if $\alpha(1+\delta^2)+\beta<1$. This has two implications:

(i) The persistence index of a NAGARCH(1,1) is $\alpha(1+\delta^2)+\beta$ and not simply $\alpha+\beta$;
(ii) a NAGARCH(1,1) model is stationary if and only if $\alpha(1+\delta^2)+\beta<1$.

3.2.10 The Absolute Value GARCH (AVGARCH)

The absolute value generalized autoregressive conditional heteroscedasticity (AVGARCH)model, according to Ali (2013), is specified as

$$a_t = \sigma_t\varepsilon_t,$$
$$\sigma^2 = \omega + \sum_{i=1}^{p}\alpha_i(|\varepsilon_{t-i}+b| - c(\varepsilon_{t-i}+b))^2 + \sum_{j=1}^{q}\beta_j\sigma_{t-j}^2 \quad (24)$$

3.3 Model Selection Criteria

The procedure for selecting the best model is by a combination of measures of goodness-of-fit of estimated GARCH models using Information Criteria, which are likelihood-based measures of model fit that include a penalty for complexity, and backtesting analysis; and the model which passes all situations during backtesting analysis, becomes selected as the best or the superior model (Adenomon *et al.*, 2022 and Emenogu, 2019).

3.3.1 Information criteria

The Information Criteria considered in this study are Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) which is also known as Schwarz Bayesian Criterion (SBIC). The GARCH model that has the lowest value of AIC and BIC/SBIC is the preferred model among competing models. They are given below as

$$AIC = -2\ln(\hat{\sigma}^2) + 2(k) - 1 - \ln(2\pi) \quad (25)$$
$$SBIC = -2\ln(\hat{\sigma}^2) + (k)*\ln(n) - 1 - \ln(2\pi), \quad (26)$$

where $\hat{\sigma}^2$ is the estimated model error variance; k is the number of parameters in the model, and n is the number of observation (Akaike, 1973 and Schwarz, 1978).

3.3.2 Half-Life volatility

Half-life volatility measures the mean reverting speed (average time) of a stock price or returns. The mathematical expression of half-life volatility is given as

$$Half-Life = \frac{\ln(0.5)}{\ln(\alpha_1+\beta_1)}. \quad (27)$$

It can be noted that the value of $\alpha + \beta_1$ influences the mean reverting speed (Ahmed *et al*., 2018), which means that if the value of $\alpha + \beta_1$ is closer to one (1), then the volatility shocks of the half-life will be longer.

3.3.3 Persistence

Volatility persistence is the strength of the volatility feedback effect. High persistence means that volatility shocks will be felt further in the future, but to a lesser extent. Determination of low or high persistence in volatility exhibited by financial time series can be by the GARCH coefficients of a stationary GARCH model. The persistence of a GARCH model can be calculated as the sum of GARCH (β_1) and ARCH (α_1) coefficients; that is, $\alpha + \beta_1$. In most financial time series, it is very close to one (1) (Banerjee & Sarkar 2006; Ahmed *et al*. 2018). Persistence could take the following conditions:

a. If $\alpha + \beta_1 < 1$: the model ensures positive conditional variance as well as stationary.
b. If $\alpha + \beta_1 = 1$: we have an exponential decay model, then the half-life becomes infinite; meaning that the model is strictly stationary.
c. If $\alpha + \beta_1 > 1$: the GARCH model is said to be non-stationary, meaning that the volatility ultimately detonates toward the infinitude (Ahmed *et al*., 2018).

In addition, the model shows that the conditional variance is unstable, unpredicted and the process is non-stationary (Kuhe, 2018).

3.3.4 Backtesting model

The unconditional (Kupiec) and conditional (Christoffersen) coverage tests used for testing the correct number of exceedances are discussed here.

The unconditional (Kupiec) test also referred to as Proportion of failure (POF)-test with its null hypothesis is given as $H_0: p = \hat{p} = \frac{y}{T}$,

where,

p is the proportion of failure (POF);
$\hat{p}$ is the sample estimate of p;
y is the number of exceptions, and
T is the number of observations.

The test is given as equation (25), using LR to denote likelihood ratio

$$LR_{POF} = -2ln\left(\frac{(1-p)^{T-y}p^y}{\left[1-(\frac{y}{p})\right]^{T-y}\frac{y}{T}^y}\right). \quad (28)$$

Under the hypothesis that the model is correct and LR_{POF} is asymptotically chi-square (χ^2) distributed with degree-of-freedom as one (1), if the value of the LR_{POF} statistic is greater than the critical value (or p < 0.01 for 1% level of significant or p < 0.05 for 5% level of significance), the null hypothesis is rejected and the model then is inaccurate. Note that p is p-value. The Christoffersen's Interval Forecast Test combined the independence statistic with the Kupiec's POF test to obtain the joint test (Christoffersen, 1998; Nieppola, 2009). This test examined the properties of a good VaR model, the correct failure rate and independence of exceptions; that is, conditional coverage (cc); the conditional coverage (cc) is given as

$$LR_{cc} = LR_{POF} + LR_{ind}, \text{ where,}$$

$$LR_{ind} = \sum_{i=2}^{n}\left[-2ln\left(\frac{p(1-p)^{u_i-1}}{(\frac{1}{u_i})(1-\frac{1}{u_i})^{u_i-1}}\right)\right] - 2ln\left(\frac{p(1-p)^{u-1}}{(\frac{1}{u})(1-\frac{1}{u})^{u-1}}\right) \quad (29)$$

where u_i is the time between exceptions i and i-1 while u is the sum of u_i. If the value of the LR_{cc} statistic is greater than the critical value (or p-value < 0.01 for 1% level of significance or p-value < 0.05 for 5% level of significance), the null hypothesis is rejected and that leads to the rejection of the model.

3.4 Distributions of GARCH Models

In this study, we employed two innovations namely student *t* and skewed student *t* distributions; they can account for excess kurtosis and non-normality in financial returns (Heracleous 2003; Wilhelmsson, 2006 and Kuhe, 2018).

The student *t* distribution is given as

$$f(y) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi}\Gamma\left(\frac{v}{2}\right)}\left(1+\frac{y^2}{v}\right)^{-\frac{(v+1)}{2}}; \qquad -\infty < y < \infty \quad (30)$$

The Skewed student *t* distribution is given as

$$f(y;\, \mu, \sigma, \nu, \lambda) = \begin{cases} bc\left(1+\frac{1}{v-2}\left(\frac{b\left(\frac{y-\mu}{\sigma}\right)+a}{1-\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & if\ y < -\frac{a}{b} \\ bc\left(1+\frac{1}{v-2}\left(\frac{b\left(\frac{y-\mu}{\sigma}\right)+a}{1+\lambda}\right)^2\right)^{-\frac{v+1}{2}}, & if\ y \geq -\frac{a}{b} \end{cases}, \quad (31)$$

where ν is the shape parameter with $2 < \nu < \infty$ and λ and is the skewness parameter with $-1 < \lambda < 1$. The constants a, b and c are given as

$$a = 4\lambda c\left(\frac{\nu-2}{\nu-1}\right); b = 1 + 3(\lambda)^2 - a^2; c = \frac{\Gamma(\frac{v+1}{2})}{\sqrt{\pi(v-2)\Gamma(\frac{v}{2})}},$$

where μ and σ are the mean and standard deviation of the skewed student *t* distribution, respectively.

4. Results and Discussion

Figure 1 plotted the daily log returns of First Bank stock price but with high spikes at the early part of the returns before the first 1000 observations. Thereafter, the returns seem to be stable over time. Figure 2 plotted the cleansed returns for outliers of the log daily returns of First Bank stock price because of the high spikes at the early part of the returns before the first 1000 observations. This is done to remove the effects of possible outliers in the return series. Table 1 presents the descriptive statistics of the log stock price and returns of First Bank of Nigeria Plc. The stock price is negatively skewed with low kurtosis value. The daily stock price is not normally distributed while the stock price is not stationary except with Phillips-Perron test that revealed stationarity. For the returns, mean value (-0.000478) which implies loss in returns, the returns is negatively skewed, with high kurtosis, not normally distributed and stationary with the presence of ARCH effects in the returns. These characteristics are typical for financial time series.

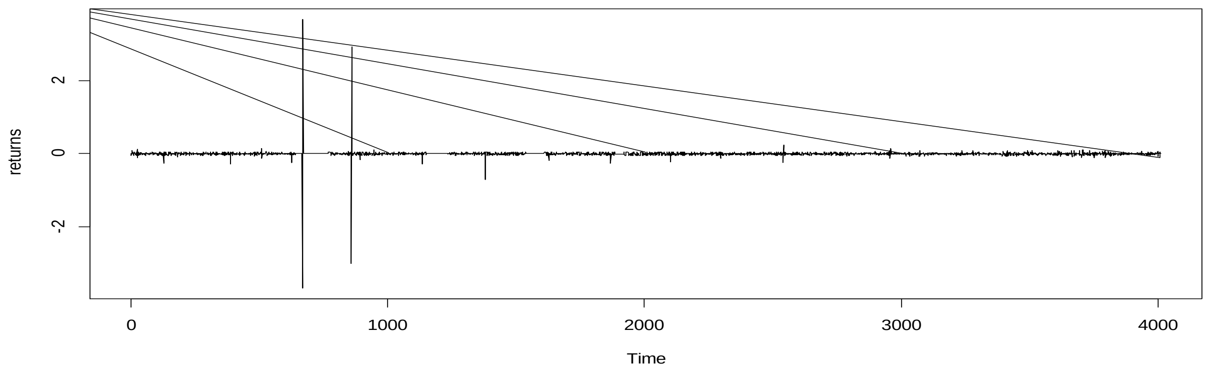

Figure 1: Plot of the Log Daily Returns of First Bank Stock Price

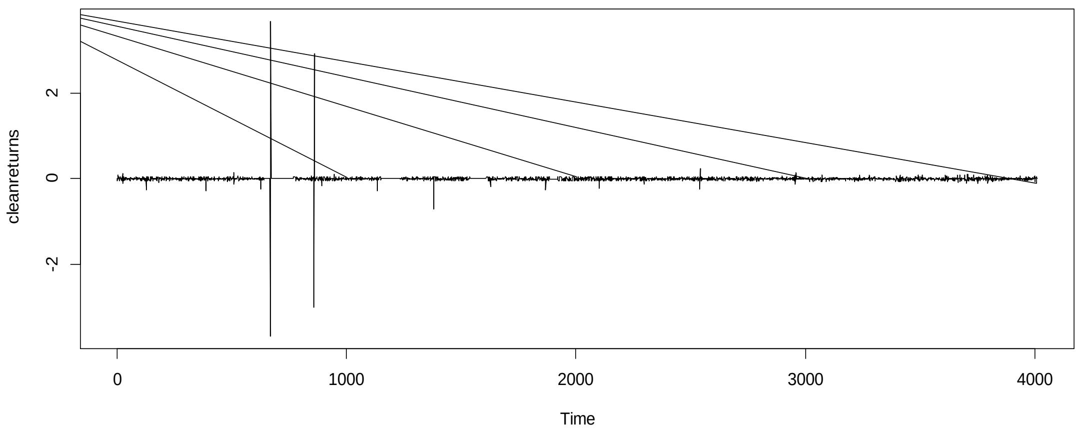

Figure 2: Plot of Cleansed Returns for Outliers of the Log Daily Returns of First Bank Stock Price

Table 1: Descriptive Statistics of the Daily Log First Bank Stock Price and Returns

Statistic	Log Stock Price	Log Stock Returns
Mean	2.846591	-0.000478
Median	2.981635	0.000000
Maximum	4.287170	3.688880
Minimum	-0.693150	-3.688880
Std. Dev.	0.694314	0.109971
Skewness	-0.883368	-0.466882
Kurtosis	3.709857	898.3602
Jarque-Bera	**606.0215**	**1.34E+08**
N	4012	4011
ADF Test	-2.393018	**-22.34187**
DF-GLS test	-1.872250	**-26.03394**
Phillips-Perron test	**-5.915994**	**-167.4472**
ARCH LM-test		**906.88**, df = 12

Note: The Bold Values are significant (p < 1%)

Table 2 presents the information criteria from each of the GARCH model. NGARCH and apARCH had very low information criteria values, but the detailed models showed poor fits. While the information criteria values for the other GARCH models are similar, the information criteria here cannot be used alone to select the superior model because of the advantage of using value-at-risk through backtesting approach. Table 3 presents the persistence and the half-life volatility values from the competing models. For both distributions, eGARCH(1,1) with student *t* distribution had the lowest values of persistence and half-life volatility. With the eGARCH(1,1) with student *t* distribution, it takes about 5 days for mean-reverting to take place in the returns.

Table 2: GARCH Models and their Performance on the Log Returns of Daily Log First Bank Returns

Model	Information criteria	Std t innovation	Skewed std t innovation
sGARCH (1,1)	Akaike	NA	NA
	Bayes		
	Shibata		
	Hannan-Quinn		
gjrGARCH(1,1)	Akaike	NA	-5.5772
	Bayes		-5.5678
	Shibata		-5.5772
	Hannan-Quinn		-5.5739
eGARCH (1,1)	Akaike	-4.8021	-5.0587
	Bayes	-4.7943	-5.0469
	Shibata	-4.8021	-5.0587
	Hannan-Quinn	-4.7993	-5.0545
iGARCH (1,1)	Akaike	-4.9317	-4.9313
	Bayes	-4.9270	-4.9250
	Shibata	-4.9317	-4.9313
	Hannan-Quinn	-4.9301	-4.9291
TGARCH(1,1)	Akaike	-5.7429	-5.7417
	Bayes	-5.7350	-5.7323
	Shibata	-5.7429	-5.7417
	Hannan-Quinn	-5.7401	-5.7384
NGARCH(1,1)	Akaike	**-15.485**	**-13.634**
	Bayes	**-15.477**	**-13.624**
	Shibata	**-15.485**	**-13.634**
	Hannan-Quinn	**-15.482**	**-13.630**
apARCH(1,1)	Akaike	**-15.154**	**-16.412**
	Bayes	**-15.144**	**-16.401**
	Shibata	**-15.154**	**-16.412**
	Hannan-Quinn	**-15.150**	**-16.408**
NAGARCH(1,1)	Akaike	-4.9276	-4.9278
	Bayes	-4.9197	-4.9184
	Shibata	-4.9276	-4.9278
	Hannan-Quinn	-4.9248	-4.9245
AVGARCH(1,1)	Akaike	-5.7378	-5.7319
	Bayes	-5.7284	-5.7209
	Shibata	-5.7378	-5.7319
	Hannan-Quinn	-5.7344	-5.7280

Note: NA-Not Available

Table 3: Persistence and Half-life Volatility of the GARCH models of daily log First Bank Stock Returns

Models	Std *t*		Skewed Std *t*	
	Persistence	Half-life volatility	Persistence	Half-life volatility
sGARCH (1,1)	NA	NA	NA	NA
gjrGARCH(1,1)	NA	NA	0.9926263	93.65594
eGARCH (1,1)	0.857391	4.505015	0.9501059	13.54285
iGARCH (1,1)	1	infinity	1	Infinity
TGARCH(1,1)	0.9484237	13.08964	0.9486083	13.1379
NGARCH(1,1)	0.9770803	29.89448	0.9662914	20.21435
apARCH(1,1)	0.9943994	123.416	NA	NA
NAGARCH(1,1)	0.9976844	298.9922	0.9974323	269.5995
AVGARCH(1,1)	0.9458538	12.45161	0.9535129	14.56119

Note: NA-Not Available

Table 4: Backtesting of the GARCH Models: GARCH Roll Forecast (Backtest Length: 2011) for the log Daily First Bank stock returns

Model	Distributions	Alpha	Expected Exceed	Actual VaR Exceed	Unconditional Coverage (Kupiec) H_0: Correct Exceedances	Conditional Coverage (Christoffersen) H_0: Correct Exceedances and independence of Failure
sGARCH(1,1)	Student t	1%	NA	NA	NA	NA
		5%	NA	NA	NA	NA
	Skewed student t	1%	NA	NA	NA	NA
		5%	NA	NA	NA	NA
gjrGARCH(1,1)	Student t	1%	NA	NA	NA	NA
		5%	NA	NA	NA	NA
		1%	10.7	31	Fail	Fail
	Skewed student t	5%	53.5	96	Fail	Fail
eGARCH(1,1)	Student t	1%	10.7	10	Pass	Pass
		5%	53.5	67	Pass	Pass
	Skewed student t	1%	10.7	10	Pass	Pass
		5%	53.5	74	Fail	Fail
iGARCH(1,1)	Student t	1%	10.7	26	Fail	Fail
		5%	53.5	88	Fail	Fail
	Skewed Student t	1%	10.7	27	Fail	Fail
		5%	53.5	91	Fail	Fail
TGARCH(1,1)	Student t	1%	10.7	36	Fail	Fail
		5%	53.5	96	Fail	Fail
	Skewed student t	1%	20.1	87	Fail	Fail
		5%	100.6	223	Fail	Fail
NGARCH(1,1)	Student t	1%	20.1	241	Fail	Fail
		5%	100.6	407	NA	NA
	Skewed student t	1%	20.1	196	Fail	Fail
		5%	100.6	347	NA	NA
apARCH(1,1)	Student t	1%	20.1	515	NA	NA
		5%	100.6	617	NA	NA
	Skewed student t	1%	NA	NA	NA	NA
		5%	NA	NA	NA	NA

Table 4 (Cont'd)

NAGARCH(1,1)	Student t	1%	20.1	61	Fail	Fail
		5%	100.6	207	Fail	Fail
	Skewed student t	1%	20.1	63	Fail	Fail
		5%	100.6	211	Fail	Fail
AVGARCH(1,1)	Student t	1%	20.1	89	Fail	Fail
		5%	100.6	225	Fail	Fail
	Skewed student t	1%	20.1	89	Fail	Fail
		5%	100.6	228	Fail	Fail

Note: NA-Not Available

It should be noted that "Not Available" (NA) occurred where the models did not converge which indicated that such models might not have been suitable. Table 4 presents the result of backtesting test of some selected competing GARCH(1,1) models in this study. The backtesting result of the sGARCH (1,1) and apARCH (1,1) models for both distributions were not available, also gjrGARCH (1,1) with student *t* distribution was not available and NGARCH (1,1) at 5% for both distributions were not available. All other GARCH models failed the backtesting except eGARCH(1,1) with student *t* distribution. Therefore, with the backtesting approach, eGARCH(1,1) with student *t* distribution emerged the superior model for modeling First Bank stock returns in Nigeria (Christoffersen, 1998; Nieppola, 2009; Kakushadze and Serur, 2018).

5. Conclusion

This paper investigated the place of backtesting approach in financial time series analysis in choosing a reliable GARCH Model for analyzing stock returns. To achieve this, a secondary data was collected from www.cashcraft.com under stock trend and analysis, and used. Daily stock prices were collected on First Bank stock price from January 2, 2001 to May 8, 2017 on 8th May, 2017. The work employed nine different GARCH models (sGARCH, gjrGARCH, eGARCH, iGARCH, aPARCH, TGARCH, NGARCH, NAGARCH and AVGARCH) with maximum lag of 1 because the work of Jafari *et al.* (2007) supported that GARCH (1,1) works well. The information criteria for the sGARCH model were not available because the model could not converge, while NGARCH and apARCH had very low information criteria values, but the detail models showed poor fits. The caution here is that GARCH model should not be selected based on only information criteria but the significance value of the coefficients, goodness-of-fit test and backtesting should be considered also (Emenogu et al. 2019 & 2020).

Results from the backtesting approach of the competing GARCH (1,1) models were explored to avoid misleading conclusion. The backtesting result of the sGARCH (1,1) and apARCH (1,1) models for both distributions were not available; also, gjrGARCH (1,1) with student *t* distribution was not available and NGARCH (1,1) at 5% for both distributions were not available. All other GARCH models failed the backtesting except eGARCH(1,1) with student *t* distribution. Therefore, with the backtesting approach, eGARCH(1,1) with student *t* distribution emerged the superior model for modeling First Bank stock returns in Nigeria (Christoffersen, 1998; Nieppola, 2009; Kakushadze and Serur, 2018). This study recommends that backtesting approach is a valuable option for financial analysts to selecting superior GARCH model for informed financial decision making.

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