Simulation study on the in-sample forecasting performances of Sims-Zha Bayesian VARX in the presence of collinearity between the exogenous variables for small sample situation

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In time series literature, exogenous variables tend to improve the forecast of the endogenous variables. This paper examined the forecast performance of six (6) versions of Bayesian Vector Autoregressive models with exogenous variables (BVARX) using normal-inverse Wishart Prior when collinearity exist between the exogenous variables for small sample situations. To achieve this, VAR(2) model was used to simulate bivariate time series from a stable process while bivariate exogenous variables were simulated from a standard normal distribution to possess the following collinearity levels: -0.99, -0.95, -0.9, -0.85, -0.8, 0.8, 0.85, 0.9, 0.95, 0.99. The experiment was carried out in R environment and repeated 10,000 times for the following time series lengths: 8, 16, 32 and 50. The Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE) were used to adjudge the models. In all the scenarios considered, BVARX4 performed best while BVARX1 performed worst in all the collinearity levels and time series lengths. Lastly, RMSE and MAE values of the BVARX models are higher with negative collinearity compared to positive collinearity while the values of RMSE and MAE for the BVARX model decreased as the time series length increased.

Keywords: BVARX; Normal-Wishart Prior; forecast; collinearity; RMSE; MAE

1. Introduction

In the field of statistics, researchers are developing methods ranging from Classical to Bayesian methods in order to gain useful inference from small samples situation so as to avoid biased and incorrect inference. In the past, some statisticians recommend non-parametric statistical methods but it has also been found that Bayesian statistics and estimation can be used to overcome the limitations peculiar to small sample sizes (van de Schoot, 2018). For example, Yahya and Olaniran (2013) compared Bayesian Linear regression against two classical regression methods (the ridge regression and ordinary least squares) on data inherent with collinearity. Their simulation study revealed the efficiency and superiority of Bayesian statistical model over the classical regression model.

In the field of econometrics, Bayesian Vector Autoregression (VAR) has shown some level of superiority over Classical VAR in terms of estimation and forecasting in small sample situation (Canova, 2007; Adenomon, 2015; Adenomon, *et al*. 2016 and Adenomon *et al*.

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2015). VAR methodology superficially resembles simultaneous equation modelling in that we consider several endogenous variables together in which each endogenous variable is explained by its lagged values and the lagged values of all the other endogenous variables in the model (Adenomon, 2017 and Gujarati, 2003). Adenomon and Oyejola (2019) examined the performance of Vector Autoregressive (VAR) and Bayesian VAR models in the presence of collinearity and autocorrelated error terms. Also, Adenomon and Oyejola (2014) studied the forecasting performances of the unrestricted VAR and Bayesian VAR in the presence of collinearity levels: 0.8, -0.8, 0.85, -0.85, 0.9, -0.9, 0.95, -0.95, 0.99 and -0.99. Similar studies are found in Adenomon *et al.* (2016, 2015). Djurovic *et al.* (2020) *investigated* the macroeconomic effects of COVID-19 in Montenegro using BVAR approach with Litterman/Minessota Prior on demand and supply losses due to illness and closed activities and their effects on GDP growth. All these previous studies did not consider the role of exogenous variables in VAR and BVAR models.

But some time series modelers believe that including exogenous variables to VAR (VARX) model improves the forecasts. It is on this background that this study implemented the Bayesian VAR with exogenous variables (BVARX) model. Recent studies in econometrics have identified the superiority of Bayesian VAR with exogenous variables (BVARX) over Bayesian VAR and standard VAR models in terms of in-sample and out-sample forecasts (Djurovic *et al*., 2020). The reason is because potential exogenous variables could improve the performance of BVAR models (Cuaresma *et al*. 2014). Application of BVARX in forecasting cryptocurrencies can be seen in Bohte and Rossini (2019) while BVARX application on the dynamic interrelationships among inflation while interest and exchange rates with the effects of money supply and GDP in Nigeria can be seen in Adenomon and Oduwole (2022).

Anttonen (2019) examined conditional BVARX forecasting model for small open economies such as Finland. In considering short term forecasting using BVARX, the study reported that BVARX model outperformed the univariate benchmark models. Burlon *et al.* (2015) considered medium-term forecasting of euro-area macroeconomic variables using DSGE, BVARX and univariate model to compare their forecasting performances. The study revealed similar performances by DSGE and BVARX models, and, DSGE and BVARX models performed more accurately than simple regression models. Ahmad and Haider (2019) compared the forecast performance of DSGE, VARX, BVARX and BVAR models on out-of-sample forecasts for GDP growth, Call money rate, CPI inflation and percent change in exchange rate in Pakistan. The models performed with respect to each endogenous variable equation (GDP growth, Call money rate, CPI inflation and exchange rate) but BVARX model provides more accurate forecast for exchange rate. Romero and Sal (2022) used BVARX model to provide evidence that inflation expectations obtained from surveys and break-even inflation measures are affected by weather supply shocks. But none of these studies investigated the effect of collinearity between the exogenous variables in BVARX model.

This study focuses on a simulation study on the in-sample forecasting performances of Sims-Zha Bayesian VARX in the presence of high collinearity between the exogenous variables for small sample situations.

2. Methodology

This study employed simulation study.

2.1 Simulation Procedure

The simulation procedure is as follows:

Step 1: Generation of an artificial two-dimensional (Bivariate data) VAR (2) process that obeys the following form:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_t = \begin{bmatrix} 5.0 \\ 7.0 \end{bmatrix} + \begin{bmatrix} 0.50.2 \\ -0.2-0.5 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} -0.3-0.7 \\ -0.10.3 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-2} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t,$$

where $u_{it} \sim N(0,1)$ for $i = 1, 2$. The choice here is similar to the work and illustration of Cowpertwait (2006).

Step 2: Ten (10) collinearity levels were considered as ρ = -0.99, -0.95, -0.9, -0.85, -0.8, 0.8, 0.85, 0.9, 0.95 and 0.99 for the exogenous variables, $X_{it} \sim N(0, 1)$ for i = 1, 2.

Step 3: Then apply the Cholesky Decomposition to the data generated in step 2 in order to create a bivariate time series data so that X_1 and X_2 have the desired correlation level.

In this present study, let the desired correlation matrix be $R = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$; then the Choleski factor, P, is $P = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1\text{-}\rho^2} \end{bmatrix}$ and the simulated data is pre-multiplied by the Choleski factor so that the simulated data is scaled to have the desired correlation level (Diebold & Mariano, 2002). The combination of steps 1 and 3 produces a bivariate time series (y_1 and y_2) and the exogenous variables (X_1 and X_2) are collinear. The simulated data assumed time series lengths (T) of 8, 16, 32 and 50.

2.2 Model Evaluation Procedure

The following procedures were used:

1. Mean Absolute Error (MAE) is given as $MAE_j = \frac{\sum_{i=1}^{n}|e_i|}{n}$. This criterion measures deviation from the series in absolute terms and measures how much the forecast is biased.
2. The Root Mean Square Error (RMSE) is given as $RMSE_j = \sqrt{\frac{\sum_i^n (y_i - y^f)^2}{n}}$, where y_i is the time series data and y^f is the forecast value of y (Caraiani, 2010).

The smaller the values of MAE and RMSE, the better the fits of the model become (Cooray, 2008). In this simulation study, $RMSE = \frac{\sum_j^N RMSE_j}{N}$ and $MAE = \frac{\sum_j^N MAE_j}{N}$, where N=10000. The model with the minimum RMSE and MAE results is the preferred model.

Table 1: Sample of simulated data for t = 8 and ρ = -0.99

y1	y2	x1	x2	
[1,]	6.3737909	11.373791	1.21269855	-0.99356972
[2,]	11.3237605	3.900453	0.35508066	-0.53343911
[3,]	2.0416691	9.033269	2.21627421	-2.23046333
[4,]	2.4013057	5.814055	-0.09054039	0.13097496
[5,]	0.3426198	9.033470	-1.31652811	1.25576384
[6,]	3.7564760	8.487529	0.06653479	0.02346259
[7,]	2.9669209	8.498111	0.51217262	-0.55123017

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[8,]     1.5096382     7.722940      0.05002974      -0.03050043

Correlation matrix from simulated data
           x1         x2
x1  1.0000000 -0.9936951
x2 -0.9936951  1.0000000
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Table 1 presents a sample of generated data for T = 8 and ρ = -0.99. The correlation from the simulated data is similar to the actual correlation coefficient.

2.3 Model Description and Specifications

2.3.1 Bayesian vector autoregression with Sims-Zha Prior and BVARX model

The BVAR model of Sims and Zha (1998) has gained popularity both in economic time series and political analysis, because the Sims-Zha BVAR allows for a more general specification and can produce a tractable multivariate normal posterior distribution, which leads to robust estimation and estimates. To construct a reduced form Bayesian SUR with the Sims-Zha prior is as follows. The prior means for the reduced form coefficients are that $B_1 = I$ and $B_2, \ldots, B_p = 0$. This means that $B_1 = I$; that is, B_1 is an identity matrix while $B_2, \ldots, B_p = 0$; that is, $B_2, \ldots B_p$ are equal to null matrices. It is assumed that the prior has a conditional structure that is multivariate Normal-inverse Wishart distribution for the parameters in the model. Again, the Sims-Zha BVAR estimates the parameters for the full system in a multivariate regression (Brandt and Freeman, 2006).

In the reduced form model,

$$y_t = c + y_{t-1}B_1 + \ .\ .\ . + y_{t-p}B_p + u_t, \tag{1}$$

where $c = dA_0^{-1}$, $B_l = -A_lA_0^{-1}$, $l = 1,2,\ldots p$, $u_t = \varepsilon_t A_0^{-1}$ and $\Sigma = {A_0^{-1}}' A_0^{-1}$. The matrix representation of the reduced form is given as

$$\underset{T\times m}{Y} = \underset{T\times(mp+1)}{X} \underset{(mp+1)\times m}{\beta} + \underset{T\times m}{U}, U \sim MVN(0, \Sigma). \tag{2}$$

The coefficients for the system of the reduced form model are estimated with the following estimators:

$$\hat{\beta} = (\Psi^{-1} + X'X)^{-1}(\Psi^{-1}\bar{\beta} + X'Y) \tag{3}$$

$$\hat{\Sigma} = T^{-1}(Y'Y - \hat{\beta}'(X'X + \Psi^{-1})\hat{\beta} + \bar{\beta}'\Psi^{-1}\bar{\beta} + \bar{S}), \tag{4}$$

where the Normal-inverse Wishart prior for the coefficients is

$$\beta/\Sigma \sim N(\bar{\beta}, \Psi) \text{ and } \Sigma \sim IW(\bar{S}, v). \tag{5}$$

This representation translates the prior form proposed by Sims and Zha from the structural model to the reduced form (Brandt and Freeman (2006, 2009) and Sims and Zha (1998, 1999)).

The procedure for BVAR with Sims-Zha prior is as follows:
Consider the following (identified) dynamic simultaneous equation model,

$$\sum_{l=0}^{p} \underset{1\times m}{y_{t-l}} \underset{m\times m}{A_l} = \underset{1\times m}{d} + \underset{1\times m}{\varepsilon_t}, \text{t} = 1,2,\ldots\text{T} \tag{6}$$

This is an m-dimensional VAR for a sample of size, T, with y_t a vector of observations at time, t, A_l is the coefficient matrix for the l^{th} lag, p is the maximum number of lags (assumed known), d is a vector of constant and ε_t, a vector of normal structural shocks such that $E[\varepsilon_t/y_{t-s}, s > 0] = \underset{1\times m}{0}$ and $E[\varepsilon_t^{'}/y_{t-s}, s > 0] = \underset{m\times m}{I}$.

The structural model can be transformed into a multivariate regression by defining A_0 as the contemporaneous conditions of the series and A_+ as a matrix of the coefficients on the lagged variables by $YA_0 + XA_+=E$ where Y is $T \times m$, A_0 is $m\times m$, X is $T\times(mp+1)$, A_+ is $(mp+1) \times m$ and E is $T\times m$ matrix. To define the VAR in a compact form,

$$a_0 = vec(A_0),\ a_+ = vec\begin{bmatrix} -A_1 \\ . \\ . \\ . \\ -A_p \\ d \end{bmatrix}, A = \begin{pmatrix} A_0 \\ A_+ \end{pmatrix},\ a = vec(A).$$

The VAR model can then be written as a linear projection of the residual by letting $Z =[Y\ X]$, and $A = [A_0/A_+]^{'}$ is a conformable stacking of the parameters in A_0 and A_+.

$$YA_0 + XA_+ = E \tag{7}$$

$$ZA = E. \tag{8}$$

In order to derive the Bayesian estimator for this structural equation model, we have to examine the (conditional) likelihood function for normally distributed residuals

$$L(Y/A) \propto |A_0|^T exp[-0.5tr(ZA)^{'}(ZA)]$$

$$\propto |A_0|^T exp[-0.5a^{'}(I \otimes Z^{'}Z)a]. \tag{9}$$

The prior overall of the structural parameters has the form, $\pi(a) = \pi(a_+/a_0)\pi(a_0)$ such that

$$\pi(a) = \pi(a_0)\varphi(\tilde{a}_+, \Psi), \tag{10}$$

$\tilde{a}_+$ denotes the mean parameters in the prior for a_+, Ψ is the prior covariance for $\tilde{a}_+$ and $\varphi(\ )$ is a multivariate normal density. The posterior for the coefficients is then

$$q(A) \propto L(Y/A)\pi(a_0)\varphi(\tilde{a}_+, \Psi) \propto \pi(a_0)|A_0|^T|\psi|^{-0.5} \times exp[-0.5(a_0^{'}(I \otimes Y^{'}Y))a_0$$

$$-2a_+^{'}(I \otimes X^{'}Y)a_0 + a_+^{'}(I \otimes X^{'}X)a_+ + \tilde{a}_+^{'}\Psi\tilde{a}_+)]. \tag{11}$$

The posterior is conditional multivariate normal since the prior has a conjugated form. In this case, the posterior can be estimated by a multivariate seeming unrelated regression (SUR) model. The forecast and inferences can be generated by exploiting the multivariate normality of the posterior distribution of the coefficients. The normal conditional prior for the mean of the structural parameters is given by $E(A_+/A_0) = \begin{bmatrix} A_0 \\ 0 \end{bmatrix}$ while $V(A_+/A_0) = \Psi$is the prior covariance matrix for $\tilde{a}_+$. Though complicated, it is specified to reflect the following general beliefs and facts about the series being model. The summary of the Sims-Zha prior is given in Table 2.

Table 2: Hyperparameters of Sims-Zha reference prior

Parameter	Range	Interpretation
λ_0	[0,1]	Overall scale of the error covariance matrix
λ_1>0		Standard deviation around A_1 (persistence)
λ_2	=1	Weight of own lag versus other lags
λ_3>0		Lag decay
λ_4	≥ 0	Scale of standard deviation of intercept
λ_5	≥ 0	Scale of standard deviation of exogenous variable coefficients

μ_5	≥ 0	Sum of coefficients/Cointegration (long-term trends)
μ_6	≥ 0	Initial observations/dummy observation (impacts of initial conditions)
v	> 0	Prior degrees of freedom

Source: *Brandt and Freeman (2006)*

The Bayesian Vector Autoregressive Model with exogenous variable is known as the BVARX (p,s) model. BVARX simply refers to a BVAR-model with suitable lag restrictions on the exogenous variables of the model (Anttonen, 2019 and Cuevas and Quilis, 2016). The form of the BVARX (p,s) model can be given as

$$y_t = \delta + \sum_{i=1}^{p} \varphi_i \, y_{t-1} + \sum_{i=0}^{s} \Theta_i \, X_{t-1} + \varepsilon_t \qquad (12)$$

where y_t is the endogenous variables, y_{t-1} is lag of the endogenous variables, X_{t-1} is lag of the exogenous variables, δ is vector of constants, φ_i is the matrix of coefficients of the lag of endogenous variables, Θ_i is the matrix of coefficients of the lag exogenous variables and ε_t is the error terms. The parameter estimates can be obtained by representing the general form of the multivariate linear model,

$$y = (X \otimes I_t)\beta + e \qquad (12)$$

The prior means for the AR coefficients are the same as those of the BVAR(p). The prior means for the exogenous coefficients are set to zero.

2.3.2 The BVARX prior

This study employed the Normal-Inverse Wishart prior. The Normal-inverse-Wishart Distribution (also known as Gaussian-inverse-Wishart distribution) is a multivariate four-parameter family of continuous probability distributions. It is the conjugate prior of a multivariate normal distribution with unknown mean and covariance matrix (the inverse of the precision matrix). Given (μ, Σ) has a normal-inverse-Wishart distribution denoted as $(\mu, \Sigma) \sim NIW(\mu_0, \lambda, \Psi, \nu)$, the probability density function (pdf) is given as $f(\mu, \Sigma / \mu_0, \lambda, \Psi, \nu) = N(\mu/\mu_0, \frac{1}{\lambda}\Sigma) W^{-1}(\Sigma/\Psi, \nu)$ (Murphy, 2007 and Adenomon and Oduwole, 2022). Here, the mean is normal while the standard deviation follows inverse Wishart distribution.

2.3.3 Model Specifications

The six (6) versions of Sims-Zha Bayesian VARX model given below:

$$\text{BVARX1} = (\lambda_0 = 0.6, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$$
$$\text{BVARX2} = (\lambda_0 = 0.8, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5)$$
$$\text{BVARX3} = (\lambda_0 = 0.6, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$$
$$\text{BVARX4} = (\lambda_0 = 0.8, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$$
$$\text{BVARX5} = (\lambda_0 = 0.9, \lambda_1 = 0.1, \lambda_3 = 1, \lambda_4 = 0.1, \lambda_5 = 0.07, \mu_5 = \mu_6 = 2)$$
$$\text{BVARX6} = (\lambda_0 = 0.9, \lambda_1 = 0.15, \lambda_3 = 1, \lambda_4 = 0.15, \lambda_5 = 0.07, \mu_5 = \mu_6 = 5),$$

where nµ is prior degrees-of-freedom given as $m+1$ where m is the number of variables in the multiple time series data. In this work, nµ is 3 (that is, two (2) time series variables plus 1(one)).

3. Results and Discussions

The data used in study were simulated in R software and analyzed using MSBVAR source code (package) in R (Brandt, 2012). Ten collinearity levels: ρ = -0.99, -0.95, -0.9, -0.85, -0.8, 0.8, 0.85, 0.9, 0.95 and 0.99 for the exogenous variables as $X_{it} \sim N(0, 1)$ for i=1,2 were considered. The experiment was repeated 10,000 times for the following time series lengths:

8, 16, 32 and 50. The Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE) were used to adjudge the models. Table 3A presents the performances of the BVARX model when collinearity levels are negative for T = 8. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014).

Table 3A: The performance of BVARX models for negative collinearity using RMSE and MAE when T = 8

	-0.99		-0.95		-0.9		-0.85		-0.8	
BVAR Models	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
BVARX	3.13599	2.08533	3.13910	2.09033	3.13314	2.08319	3.13315	2.08208	3.12540	2.07730
1	8	1	9	7	3	7	3	0	0	3
BVARX	3.11404	2.08612	3.08705	2.06506	3.11815	2.09207	3.10059	2.07858	3.10190	2.07872
2	8	7	5	8	7	3	4	2	9	1
BVARX	2.92336	2.03478	2.93294	2.04540	2.93328	2.04296	2.92827	2.04072	2.93435	2.04589
3	5	0	2	1	3	7	2.79795	1.98603	7	8
BVARX	2.81751	2.00184	2.80840	1.99316	2.80709	1.98933	9	2	2.79657	1.98353
4	9	2	1	3	0	1	2.91888	2.03386	4	9
BVARX	2.91743	2.03347	2.91756	2.03116	2.91845	2.03453	8	8	2.91385	2.02914
5	3	5	8	4	8	4	3.05059	2.07754	5	2
BVARX	3.05924	2.08666	3.06211	2.08687	3.05622	2.08671	2	5	3.05772	2.08534
6	6	5	8	4	6	4			9	5

Table 3B: The Ranks of the performance of BVARX models for negative collinearity using RMSE and MAE when T = 8

BVAR Models	**-0.99**		**-0.95**		**-0.9**		**-0.85**		**-0.8**	
	RMSE	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
BVARX1	6	4	6	6	6	4	6	6	6	4
BVARX2	5	5	5	4	5	6	5	5	5	5
BVARX3	3	3	3	3	3	3	3	3	3	3
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	2	2	2	2	2	2	2	2
BVARX6	4	6	4	5	4	5	4	4	4	6

Table 3B presents the ranks of the performances of the BVARX model when collinearity levels are negative for T = 8. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019).

Table 4A: The performance of BVARX models for positive collinearity using RMSE and MAE when T = 8

BVAR Models	**0.8**		**0.85**		**0.9**		**0.95**		**0.99**	
	RMSE	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**	**RMSE**	**MAE**
BVARX1	3.130429	2.079686	3.120308	2.073409	3.129439	2.079585	3.128001	2.080009	3.139624	2.088267
BVARX2	3.107252	2.085717	3.111771	2.084990	3.091810	2.070154	3.111621	2.086553	3.100658	2.077432
BVARX3	2.932088	2.044029	2.936135	2.046646	2.933603	2.043277	2.930062	2.039985	2.926647	2.037369
BVARX4	2.801023	1.986722	2.803246	1.990417	2.812059	1.996623	2.802176	1.986436	2.802425	1.987734
BVARX5	2.914694	2.030928	2.903678	2.020362	2.917399	2.032837	2.928287	2.042207	2.907373	2.026480

BVARX6	3.052600	2.079622	3.051955	2.082724	3.068258	2.095702	3.062436	2.088269	3.059850	2.087281

Table 4A presents the performances of the BVARX model when collinearity levels are positive for T = 8. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014). Table 4B presents the ranks of the performances of the BVARX model when collinearity levels are positive for T = 8. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019).

Table 4B: The Ranks of the performance of BVARX models for positive collinearity using RMSE and MAE when T = 8

BVAR	0.8		0.85		0.9		0.95		0.99	
Models	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	5	6	4	6	5	6	4	6	6
BVARX2	5	6	5	6	5	4	5	5	5	4
BVARX3	3	3	3	3	3	3	3	2	3	3
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	2	2	2	2	2	3	2	2
BVARX6	4	4	4	5	4	6	4	6	4	5

Table 5A: The performance of BVARX models for negative collinearity using RMSE and MAE when T=16

	-0.99		-0.95		-0.9		-0.85		-0.8	
BVAR Models	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	2.812930	2.031671	2.798588	2.018672	2.813643	2.033646	2.807245	2.023778	2.811811	2.030606
BVARX2	2.760303	2.004857	2.786362	2.022474	2.773796	2.014619	2.775748	2.014898	2.772198	2.013332
BVARX3	2.585100	1.919529	2.590900	1.926203	2.596321	1.929495	2.588953	1.925736	2.583053	1.919715
BVARX4	2.454330	1.846693	2.459890	1.851712	2.451674	1.842699	2.460181	1.850194	2.46243	1.85481
BVARX5	2.579854	1.916666	2.584833	1.923105	2.586140	1.920627	2.586319	1.922999	2.580663	1.918171
BVARX6	2.693967	1.980039	2.705235	1.990007	2.701032	1.984787	2.709144	1.994021	2.697146	1.981430

Table 5A presents the performances of the BVARX model when collinearity levels are negative for T=16. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014).

Table 5B: The Ranks of the performance of BVARX models for negative collinearity using RMSE and MAE when T=16

BVAR	-0.99		-0.95		-0.9		-0.85		-0.8	
Models	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	6	6	5	6	6	6	6	6	6
BVARX2	5	5	5	6	5	5	5	5	5	5
BVARX3	3	3	3	3	3	3	3	3	3	3
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	2	2	2	2	2	2	2	2
BVARX6	4	4	4	4	4	4	4	4	4	4

Table 5B presents the ranks of the performances of the BVARX model when collinearity levels are negative for T=16. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019). Table 6A presents the performances of the BVARX model when collinearity levels are positive for T=16. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014).

Table 6A: The performance of BVARX models for positive collinearity using RMSE and MAE when T=16

BVAR Models	0.8		0.85		0.9		0.95		0.99	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	2.798924	2.019183	2.806624	2.026992	2.809248	2.027360	2.806848	2.024647	2.804263	2.023614
BVARX2	2.766266	2.007922	2.770482	2.012697	2.783641	2.019675	2.779531	2.019956	2.784787	2.021780
BVARX3	2.582682	1.917906	2.574136	1.911195	2.588722	1.922973	2.577769	1.915393	2.594541	1.927690
BVARX4	2.460256	1.852220	2.472832	1.864006	2.454016	1.845401	2.452920	1.844403	2.456687	1.848319
BVARX5	2.570291	1.910370	2.593510	1.929127	2.587335	1.922978	2.584202	1.920864	2.571691	1.908511
BVARX6	2.705103	1.988580	2.700302	1.984756	2.705028	1.989710	2.706146	1.992228	2.700475	1.985611

Table 6B: The Ranks of the performance of BVARX models for positive collinearity using RMSE and MAE when T=16

BVAR Models	0.8		0.85		0.9		0.95		0.99	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	6	6	6	6	6	6	6	6	6
BVARX2	5	5	5	5	5	5	5	5	5	5
BVARX3	3	3	2	2	3	2	2	2	3	3
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	3	3	2	3	3	3	2	2
BVARX6	4	4	4	4	4	4	4	4	4	4

Table 6B presents the ranks of the performances of the BVARX model when collinearity levels are positive for T = 8. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019).

Table 7A: The performance of BVARX models for negative collinearity using RMSE and MAE when T=32

BVAR Models	-0.99		-0.95		-0.9		-0.85		-0.8	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	2.573611	1.945541	2.575013	1.946376	2.572503	1.945797	2.577942	1.950560	2.574886	1.944586
BVARX2	2.529589	1.919738	2.535785	1.925813	2.528776	1.921877	2.530915	1.923519	2.534221	1.927142
BVARX3	2.294212	1.768976	2.287775	1.762924	2.290950	1.766869	2.291899	1.767804	2.282951	1.759849
BVARX4	2.136782	1.659607	2.135215	1.658982	2.134008	1.657419	2.132074	1.655881	2.138255	1.661601
BVARX5	2.284004	1.761010	2.281137	1.759729	2.281948	1.758492	2.282447	1.759653	2.289162	1.765569
BVARX6	2.409389	1.847613	2.406261	1.844927	2.422617	1.856242	2.419723	1.855705	2.417804	1.854783

Table 7A presents the performances of the BVARX model when collinearity levels are negative for T=32. There are evidences that the RMSE and MAE values fluctuate for the

BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014). Table 7B presents the ranks of the performances of the BVARX model when collinearity levels are negative for T=32. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019).

Table 7B: The Ranks of the performance of BVARX models for negative collinearity using RMSE and MAE when T=32

BVAR Models	-0.99		-0.95		-0.9		-0.85		-0.8	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	6	6	6	6	6	6	6	6	6
BVARX2	5	5	5	5	5	5	5	5	5	5
BVARX3	3	3	3	3	3	3	3	3	2	2
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	2	2	2	2	2	2	3	3
BVARX6	4	4	4	4	4	4	4	4	4	4

Table 8A: The performance of BVARX models for positive collinearity using RMSE and MAE when T=32

BVAR Models	0.8		0.85		0.9		0.95		0.99	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	2.569286	1.942331	2.577556	1.948702	2.569850	1.942587	2.577333	1.949878	2.567758	1.941346
BVARX2	2.53216	1.92364	2.532265	1.922695	2.533177	1.923521	2.537227	1.926659	2.534564	1.925990
BVARX3	2.286717	1.763284	2.299468	1.772855	2.286785	1.761223	2.287450	1.763609	2.283513	1.760712
BVARX4	2.135035	1.657249	2.131751	1.656430	2.135563	1.659404	2.133758	1.656614	2.138753	1.660388
BVARX5	2.281912	1.758540	2.276432	1.756494	2.284687	1.761507	2.284790	1.760876	2.290296	1.766157
BVARX6	2.416885	1.854094	2.406085	1.845153	2.419983	1.856581	2.414366	1.850101	2.414263	1.851353

Table 8A presents the performances of the BVARX model when collinearity levels are positive for T=32. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014).

Table 8B: The Ranks of the performance of BVARX models for positive collinearity using RMSE and MAE when T=32

BVAR Models	0.8		0.85		0.9		0.95		0.99	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	6	6	6	6	6	6	6	6	6
BVARX2	5	5	5	5	5	5	5	5	5	5
BVARX3	3	3	3	3	3	2	3	3	2	2
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	2	2	2	3	2	2	3	3
BVARX6	4	4	4	4	4	4	4	4	4	4

Table 8B presents the ranks of the performances of the BVARX model when collinearity levels are positive for T=8. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019). Table 9A presents the performances of the BVARX model when collinearity levels are negative for T=50. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014).

Table 9A: The performance of BVARX models for negative collinearity using RMSE and MAE when T=50

BVAR Models	-0.99		-0.95		-0.9		-0.85		-0.8	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	2.464633	1.900409	2.462487	1.899269	2.469692	1.904271	2.469753	1.905415	2.462691	1.898388
BVARX2	2.412594	1.868663	2.412341	1.868414	2.412919	1.868585	2.410355	1.867142	2.410524	1.866744
BVARX3	2.104866	1.644709	2.100592	1.642310	2.101780	1.642127	2.102783	1.643771	2.103168	1.644327
BVARX4	1.929297	1.513688	1.928439	1.513507	1.928123	1.512737	1.930149	1.514230	1.928861	1.513064
BVARX5	2.100805	1.641145	2.096969	1.639120	2.100839	1.640332	2.101526	1.641708	2.099157	1.639830
BVARX6	2.248217	1.752875	2.244509	1.749916	2.249148	1.753661	2.246784	1.751755	2.251163	1.755105

Table 9B: The Ranks of the performance of BVARX models for negative collinearity using RMSE and MAE when T=50

BVAR Models	-0.99		-0.95		-0.9		-0.85		-0.8	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	6	6	6	6	6	6	6	6	6
BVARX2	5	5	5	5	5	5	5	5	5	5
BVARX3	3	3	3	3	3	3	3	3	3	3
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	2	2	2	2	2	2	2	2	2	2
BVARX6	4	4	4	4	4	4	4	4	4	4

Table 9B presents the ranks of the performances of the BVARX model when collinearity levels are negative for T=50. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019).

Table 10A: The performance of BVARX models for positive collinearity using RMSE and MAE when T=50

BVAR Models	0.8		0.85		0.9		0.95		0.99	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	2.466963	1.902444	2.468374	1.903039	2.471223	1.906307	2.465040	1.900589	2.467097	1.903920
BVARX2	2.412528	1.867885	2.408923	1.865802	2.414330	1.870976	2.411667	1.866118	2.416476	1.871287
BVARX3	2.103166	1.644145	2.103190	1.643362	2.098381	1.640549	2.102128	1.643800	2.102837	1.643264
BVARX4	1.930646	1.514738	1.927729	1.512934	1.927781	1.513071	1.927721	1.512451	1.930611	1.515146
BVARX5	2.103419	1.644219	2.101458	1.642388	2.101071	1.642288	2.098534	1.640110	2.099307	1.640598
BVARX6	2.247231	1.751503	2.251874	1.755360	2.245004	1.750137	2.246979	1.750223	2.249725	1.754567

Table 10A presents the performances of the BVARX model when collinearity levels are positive for T=50. There are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased (Adenomon and Oyejola, 2014). Table 10B presents the ranks of the performances of the BVARX model when collinearity levels are positive for T=50. At all levels of collinearity, BVARX4 was superior while BVARX1 was worst using RMSE and MAE values (Anttonen, 2019). Lastly, the values of the RMSE and MAE for the BVARX models reduced as the time series length increased which is similar to reality (Adenomon et al. 2015 and 2016).

Table 10B: The Ranks of the performance of BVARX models for positive collinearity using RMSE and MAE when T=50

BVAR Models	0.8		0.85		0.9		0.95		0.99	
	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE	RMSE	MAE
BVARX1	6	6	6	6	6	6	6	6	6	6
BVARX2	5	5	5	5	5	5	5	5	5	5
BVARX3	2	2	3	3	2	2	3	3	3	3
BVARX4	1	1	1	1	1	1	1	1	1	1
BVARX5	3	3	2	2	3	3	2	2	2	2
BVARX6	4	4	4	4	4	4	4	4	4	4

4. Conclusion

This paper examined the forecast performance of six (6) versions of Bayesian Vector Autoregressive models with exogenous variables (BVARX) using normal-inverse Wishart prior when collinearity exist between the exogenous variables for small sample situations. The BVARX models where denoted as BVARX1, BVARX2, BVARX3, BVARX4, BVARX5 and BVARX6. To achieve this, VAR(2) model was used to simulate bivariate time series from a stable process while bivariate exogenous variables were simulated from a standard normal distribution to possess the following collinearity levels (-0.99, -0.95, -0.9, -0.85, -0.8, 0.8, 0.85, 0.9, 0.95, 0.99). The experiment was carried out in R environment and repeated 10,000 times for the following time series lengths (8, 16, 32 and 50). The Root Mean Square Error (RMSE) and the Mean Absolute Error (MAE) were used to adjudge the models. In all the scenarios considered, there are evidences that the RMSE and MAE values fluctuate for the BVARX model as the collinearity levels increased while BVARX4 performed best while BVARX1 performed worst in all the collinearity levels and time series length. Lastly, RMSE and MAE values of the BVARX models are higher with negative collinearity compared to positive collinearity while the values of RMSE and MAE for the BVARX model decreased as the time series length increased.

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